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GEOMETRIC SECTIONING TASKS AND GEOMETRY ACHIEVEMENT

BY

© DALE R. DROST

A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "Geometric Sectioning Tasks and Geometry Achievement" submitted by Dale R. Drost in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Abstract

The two purposes of this study were to provide an indepth analysis of the ability of students to section solids and to investigate the relationship between this ability and geometry achievement. Two forms of a sectioning test were developed and administered to 38 classes of students in grades 5 to 10 in the Edmonton area. The first form was administered just prior to each class's regularly scheduled geometry unit and the second form just after the unit.

For the first purpose a 6 x 2 x 3 grade by sex by ability design was used with a stratified random sample of 432 subjects. The results indicated that by grade 7 most students could successfully section solids involving all cuts except the oblique cut. The transverse cut on the cone also caused difficulties. Some sections were more difficult to represent by drawings than by selection from a set of distractors. These included sections which were equilateral, parallelograms, or ellipses. The Van Hiele theory was used to help explain these difficulties.

Sectioning ability appears to be very complex. Some sections appear to require a unique ability to recognize while other possible factors include the shape of the section, the solid, and perhaps the cut. Whatever the factorial construction of the ability, the ability to section solids increased with grade level, males consistently

scored higher than females, and the high ability students scored higher than those of average ability who in turn did better than low ability students. The sex difference was particularly noticeable at the low ability level. Large increases on scores on the sectioning tests occurred with average ability students between grades 5 and 6 and with students of low ability between grades 8 and 9.

For the second purpose, two situations were identified at each grade level. Each consisted of all students in one class, or in a group of classes which studied the same geometry under similar conditions. The sectioning tests were found to be useful in predicting geometry achievement in grades 5, 7, and 8, particularly in grade 8 where the content was mainly the study of motion geometry. In grades 6, 9, and 10 the tests were of limited use in predicting achievement, both by themselves, and when considered with other possible predictors.

The results of the study suggested many possibilities for future research, particularly in the factorial construction of sectioning ability and in the use of the sectioning tests for predicting geometry achievement in certain situations.

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CHAPTER I

The Problem

Background to the Problem

Geometry has formed an integral part of the mathematics curriculum since the time of Plato. Since that time the content of geometry and the approach taken to it have remained relatively static. The content continues to be the geometry of Euclid, the method of teaching continues to emphasize the process of deduction, and the high school continues to be the main site for the teaching of geometry.

During the 1960's major reports such as, Goals for School Mathematics, The Report of the Cambridge Conference on School Mathematics (1963) in the U.S.A., Mathematics in the Primary School (1965) in England, and Geometry, Kindergarten to Grade 13, Report of the (K-13) Geometry Committee (1967) in Canada recommended changes in the role of geometry within the mathematics curriculum. More geometry was advocated throughout the school program, particularly during the elementary and junior high years. Each of these reports urged that the approach to geometry during these early years be highly intuitive and involve extensive use of concrete materials. The content throughout the school program, although still based heavily on traditional Euclidean concepts, would include material from geometric approaches such as coordinates, vectors,

transformations and elementary topology.

In the 1970's the dialogue continues to focus on what geometry should be included in the curriculum, what pedagogical techniques are most appropriate for the teaching of geometry, and what prerequisite behaviors are necessary in order that children might experience success with geometric concepts. The 36th yearbook of the National Council of Teachers of Mathematics, Geometry in the Mathematics Curriculum (Henderson, 1973), provides insight into both informal and formal approaches to geometry from kindergarten to grade 14. Many of the approaches to geometry, as well as the methods of teaching, are similar to those recommended in the reports mentioned above.

A recent workshop was held at the University of Georgia to stimulate dialogue among mathematics educators with the objective of synthesizing existing knowledge concerning the child's conception of space and geometry. The report of this workshop, Space and Geometry (Martin, 1976d), contains a wealth of information about how children, particularly those of elementary school age, perceive spatial and geometrical concepts. After reflecting on the papers presented at the workshop, Martin remarked on how little we know about the child's conception of space, despite this wealth of available information. Since a firm foundation in spatial concepts is a prerequisite for study in geometry, the necessity of extending our knowledge of the child's conception of space is of immediate importance.

The aims of the study of geometry are perhaps best put

forth in the Cambridge report (1963).

> To develop the planar and spatial intuition of the pupil, to afford a source of visualization for arithmetic and algebra, and to serve as a model for that branch of natural science which investigates physical space by mathematical methods. (p. 33, underlining mine)

The development and use of spatial abilities are emphasized by these aims. This emphasis has been put forth by eminent mathematics educators throughout the past 10 years. Dienes and Golding (1966) define geometry as the exploration of space. Adler (1968) considers geometry as the mathematical model of physical space. Fehr (1972) sees one role of geometry to be to transmit information about space and Fawcett (1970) states his first major objective of a geometry program as "geometric literacy and competence in using the basic facts of geometry to explore the spatial relationships of two- and three-dimensional figures." Martin (1976d) expresses the opinion that the instruction of geometry in the elementary school should be aimed at assisting the child to develop a well organized concept of space. Similar statements could be made concerning the junior and senior high geometry programs. There can be little doubt of the intimate relationship existing between the study of geometry and the study of space.

The impetus for much of the recent work on children's ability to perform fundamental operations in space and geometry, including the Georgia workshop, lies in the two

publications, The Child's Conception of Space (Piaget and Inhelder, 1967) and The Child's Conception of Geometry (Piaget, Inhelder, and Szeminska, 1960). The child's ability to perform particular tasks in topological space, projective space, and in Euclidean space is examined in detail in these two books. Of interest in this study is Piaget's experiment on sectioning solid figures (Piaget and Inhelder, 1967). Piaget and Inhelder claim that the ability to represent the sections of solid figures is an indication of the emergence of the ability to operate in both projective and Euclidean space. Although a variety of approaches are presently used in the instruction of geometry, all rely almost entirely on the Euclidean properties of the geometric objects studied. The emergence of the ability to operate in Euclidean space is very relevant to the study of geometry throughout the school mathematics program.

Statement of the Problem

Despite the general agreement as to the importance of spatial relationships in the study of geometry, the importance of the study of geometry, and the abundance of research available indicating that children can experience success with a wide variety of geometrical situations (Williford, 1972), students and teachers alike often express difficulty with geometry, dislike of geometry, or both. There seems to be a general opinion among mathematics educators that elementary school teachers either omit geometry or introduce it only if there is extra time

(Trafton and LeBlanc, 1973). Similar statements could well be made of secondary school teachers. Gearhart (1975) reports that in the opinions of teachers, many high school students do not experience success with geometry and do not enjoy the subject. Whether the concentration in the high school programs is on the traditional Euclidean approach, a transformational approach, a cartesian or vector approach, or some other approach to the study of geometry, success depends on the ability of the elementary and junior high schools to provide a firm background of experiences in Euclidean space. At present the programs in these areas do not appear to be meeting this need adequately.

To bring about improvement in the teaching of geometry, research is needed which will provide classroom teachers with more information as to how children learn geometry. The problem considered in this study was to investigate a particular spatial ability, that of sectioning solid figures, and to determine what relationship exists between this ability and achievement in geometry.

Definitions

A <u>solid</u> is a model of a three-dimensional object. The solids employed in this study were the cube, triangular prism, parallelepiped, cone, rectangular prism, cylinder, star, and square pyramid. These solids were constructed of cardboard and are described in detail in Chapter III.

A <u>section</u> is the plane figure formed by the intersection of a plane with a geometric solid.

A cut is the orientation of the plane which intersects the solid. The cuts employed in the present study were identified as longitudinal, transverse, parallel, and oblique and are described in detail in Chapter III.

The act of sectioning a solid is the representation of the section resulting from a cut on a solid with a drawing of the plane figure or by selecting a drawing of the section from a set of five options.

Purposes of the Study

The major purposes of this study were two-fold. The first of these was to provide an indepth analysis of the ability of students in grades 5 through 10 to section solids. A testing instrument was developed based on the research of Piaget and Inhelder (1967) and on the related work of Dodwell (1961), Lovell (1962, 1971), Boe (1966), Davis (1969), Bober (1973) and Pothier (1975). Considerable information on the ability of subjects of a particular sex, age, and ability to perform various sectioning tasks is available in these reports. They will be reviewed in detail later in this report. This study was designed to investigate these factors using carefully redesigned tasks administered in a group setting to a large sample of students from grades 5 to 10.

The results of the research cited above indicate that some sectioning tasks are much more difficult to perform than others. For example, subjects of all ages experienced difficulty on all solids with tasks involving the oblique

cut. It is possible that the ability to identify sections which result from an oblique cut is different from the ability to identify sections resulting from other cuts. Two methods of response were used in several of the sectioning experiments cited above. These were drawing the section and selecting the correct section from a group of options. The data from the Pothier study were analyzed and the possibility that the ability to draw might be an important factor in experiencing success on the sectioning tasks was indicated by the results. In this study the factor structure of sectioning ability was explored and an attempt was made to determine if the ability to section solids is a uni-factor trait or if there are several underlying factors present. Underlying factors might include drawing ability or ability to recognize sections resulting from a particular cut such as the oblique cut.

The second major purpose of this study was to investigate the relationship between the ability to section solids and achievement in geometry. The use of the testing instrument on sectioning solids as a predictor of achievement in geometry was compared to the use of other predictors such as sex, age, previous year's mathematics grade, last mathematics test score, and IQ. Each of these predictors was readily available to the classroom teachers participating in the study. This aspect of the study was also exploratory in nature. The predictive validity of the sectioning test was evaluated at each grade level used in the study. A variety of geometric settings were investigated as they

occurred in existing classrooms. The purpose here was to provide information which could prove valuable to the classroom teacher in determining which students might be expected to experience difficulty in learning geometry.

These two purposes are summarized in the four research questions listed below.

1. How do students respond to sectioning tasks involving particular cuts and solids?

2. Are there differences in the ability of students of a particular sex, grade level, and ability to section solids?

3. Is the ability to section solids a uni-factor trait or is it a composite of several independent or related abilities?

4. Is a test on sectioning solids useful to the classroom teacher for predicting achievement in geometry? A closely related question is: Does a test on sectioning solids contribute to other predictors which are readily available to the classroom teacher?

Need for the Study

Lovell (1971) writes, "There is little doubt that spatial work has in the past been neglected in the education of small children." Dodwell (1971), while commenting on the firmness of the empirical base for Piaget's statements concerning children's understanding of geometry and spatial relations, states that "Very little attention has been paid to children's perceptions and their geometrical notions." While commenting on the mathematical abilities of students

entering a Canadian university, Coleman, Edwards and Beltzer (1975) note "A lamentable weakness under the old and new regime was and is their lack of three-dimensional spatial ability." These statements taken together with similar statements referred to earlier in this report strongly suggest a need for research into the area of spatial relations, particularly as it relates to the study of geometry.

A report supported by the National Science Foundation in the United States, <u>Overview and Analysis of School Mathematics Grades K-12</u> (1975), states that "Though geometry is mentioned as being parts of texts, objectives, and testing, 78% of the teachers report spending fewer than 15 class periods per year on geometry topics." Their conclusion is that the acceptance of geometry in elementary and junior high programs has been slow indeed. Similar conclusions may be arrived at from observations of the results of National Assessment of Educational Progress reports (Carpenter, Coburn, Reys, and Wilson, 1975a, 1975b). Only 45% of 17 year olds could recall the name of a cube while only 74% could identify the shape of a cube. The results for younger children and other shapes were considerably lower. The conclusions of these reports also suggest that basic concepts of length, area and volume are not well understood. A need certainly exists to provide teachers with more information on abilities prerequisite to the learning of geometry concepts.

Considerable research has already been conducted on

the ability to section solids, as mentioned in the previous section. Yet, of these studies, only that of Bober (1973) had a direct connection to the study of geometry and to the classroom in general.

Graduate students have been accused of conducting research that is fragmentary, uncoordinated and of low impact (Clifford, 1973). As a result classroom teachers see most research as being of little direct value to them. In this study an attempt was made to bring together the results of previous work in a given area and to build upon those results. The design of the test was improved upon by maintaining consistency in the positioning of the solids and the direction of the cuts. A group method of testing was used and each student had a set of multiple choice distractors of his own, whereas previous group testing experiments had used the overhead projector. Finally, an attempt was made to determine the relationship of the test to achievement in geometry as it was taught in existing classrooms.

The need for this study can be summarized by the following two quotations.

> Especially in geometry, children make mathematical judgements using qualitatively different methods than those typically used by adults. Yet the nature of these differences is not clearly understood. Consequently, research concerning the evolution of spatial concepts would help mathematics educators better understand the difficulties that are implicitly involved in a wide range of mathematical concepts. (Lesh, 1976, p.186)

> Applications of an underlying theory of the child's conception of space or the child's conception of geometry are minimal. (Martin, 1976d, p.1)

In this study the body of knowledge concerning the ability of students to section geometric solids was extended and applied in selected classrooms where geometry was being taught.

Delimitations of the Study

1. The study was delimited to students enrolled in grades 5 through 10 in the Edmonton Public and Separate School systems.

2. The study was delimited to the particular solids and cuts selected for use on the testing instruments.

3. There exist countless possibilities for the selection of both content and teaching method for a unit on geometry. With regard to the purpose of investigating the predictive validity of the sectioning test, this study was delimited to the geometry content and to the teaching methods used by the teachers participating in the study.

Limitations of the Study

A significant feature of the study was the use of the group method of testing. The use of this method allowed a large number of subjects to be tested in a short period of time, therefore it was more efficient than an individual method of testing. However, several assumptions must be made when using the group method. It was assumed that after the short preliminary instructions, each individual student understood the nature of the task he was asked to perform. Each student was able to view each sectioning task performed a minimum of three times. It was assumed that every student

was able to make at least one observation from the desired direction. These directions are described in detail in Chapter III.

The tests were administered in 38 classrooms over a period of 7 months. It was assumed that the conditions existing in a classroom at the time of testing, such as time of day, time of year, lighting, or seating arrangements did not affect the results. Although the tester generally followed strict protocols in administering the test, it was assumed that any flexibility allowed for in the protocols did not influence the responses of the students. The protocols are presented in Appendix 1 and are discussed further in Chapter III.

The subjects were required to draw the sections on one portion of the test. It was assumed that the subjects were able to represent the sections they perceived by a drawing. This required the subject to draw what he believed the section to be. For example, if he thought the section was a square, it was assumed he could draw a square. Scoring the drawing responses imposed further limitations. Rigid criteria were composed for each drawing and it was assumed that if a drawing met these criteria, it was an indication that the subject knew the correct section.

For the purpose of investigating the predictive validity of the sectioning test, intact classes were used in the study. At least two classes were tested at each grade level where the content and teaching techniques were similar. Even in these cases the size of the sample was small for the purpose

of analysis. Although this aspect of the study was considered to be exploratory in nature, the small samples constitute a limitation of the study.

The tests used to measure achievement in geometry were constructed by the teacher. Also measures of the previous year's final mathematics grade and the last mathematics test score were collected from the cumulative records and the teachers' records respectively. It was assumed that these tests were valid and reliable measures of achievement. Although this must be viewed as a limitation due to the statistical procedures employed, the teacher-made tests were the only valid ones to use if the information obtained was to be applicable to "real" classrooms. Lorge-Thorndike IQ scores were also collected from the cumulative records and it was assumed that these scores were indicative of the cognitive ability of the subjects.

Outline of the Report

The statement of the problem has been presented in this first chapter. Chapter II contains a discussion of the literature related to the problem. The design, administration, and scoring of the sectioning tests are discussed in Chapter III. The design of the study, description of the sample, and the hypotheses tested are presented in Chapter IV. It also includes a brief description of the methods of analyses employed in the study. The results are stated in Chapter V. The final chapter consists of a summary of the study, a discussion of the results, and implications for education and for future research.

CHAPTER II

Review of the Literature

The ability of youngsters to section geometric solids was first investigated by Piaget and his associates over 30 years ago (Piaget and Inhelder, 1967). In this chapter, Piaget's theory of development is discussed as it relates to the child's conception of space and geometry. The Van Hiele theory on levels of thought in geometry is briefly discussed as an alternative to Piaget's theory as it relates to the sectioning tasks. The research dealing with the sectioning of geometric solids is reviewed and the results of these studies are compared and contrasted. The research on prediction of achievement in geometry as it relates to the present study is examined. This literature is related to form a framework to help answer the questions of Chapter I.

Piaget and the Child's Conception of Space

Most recent research basic to the study of spatial and geometric concepts has had its roots in the work of Piaget. The major themes of Piaget's studies in the areas of space and geometry have been put forth by Kidder (1976, p. 41).

> First, Piaget's major focus in space development is on space representation, not space perception. Second, he believes that those spatial representations are built up through the organization of mental "actions" performed on objects in space. And third, he claims that the child's earliest spatial notions are topological in nature and that his projective and Euclidean concepts are concomitant extensions of those topological concepts.

Smock (1976) listed similar themes while extending the organization of mental "actions" to include the role of logico-mathematical experience in the building up of spatial representation. Flavell's (1963) summary of the three "leitmotifs" which run through The Child's Conception of Space (Piaget and Inhelder, 1967) are in agreement with the themes suggested by Kidder and Smock.

The distinction between space representation and space perception is made clear by Piaget and Inhelder (1967).

> The evolution of spatial relations proceeds at two different levels. It is a process which takes place at the perceptual level and at the level of thought or imagination.... (p. 3)

> Perception is the knowledge of objects resulting from direct contact with them. As against this, representation or imagination involves the evocation of objects in their absence or, where it runs parallel to perception, in their presence. (p. 17)

Kidder (1977) stated that the perceptual level is based on sensory impressions such as feeling and seeing. Representational space, on the other hand, both extends and benefits from perception. The child is able to perform

mental operations on objects that can be only imagined, as well as objects that are present. Mental imagery is particularly adapted to spatial representation (Montangero, 1976). However, the final representation is the result of a long and arduous developmental construction which is more dependent upon actions than upon perception per se (Flavell, 1963).

Spatial representation evolves through the organization of actions performed on objects in space, beginning with sensorimotor actions, and moving later to internalized actions which eventuate in operational systems (Flavell, 1963). This developmental aspect of representation suggests that the representative space of adults differs from that of children. This is due to the vast advantage an adult possesses with regard to experience in actively manipulating the spatial environment (Martin, 1976b).

Piaget (1964) also makes a distinction between the figurative and operative aspects of knowing. The figurative aspects deal essentially with fixed states independent of any transformations and include perception, imitation, and mental imagery. The operative function involves physical transformations of objects as well as interiorized mental operations. Montangero (1976) notes that the elementary forms of operative knowing are concrete operations while the more advanced forms are mental operations with group-like structures.

In the present experiment, each subject observed the tester holding a geometric solid and watched him illustrate

a hypothetical cut on that solid. The subject had to imagine, or form a mental image of the resultant section, perform a mental operation on this representation, and then draw a copy of this image in his answer booklet. Alternately he had to select a copy of his image from a set of distractors. The task of sectioning solids thus required each subject to operate in representational space using interiorized actions.

Since the geometric solid was not actually cut, the section itself could not have been envisioned by operating only in perceptual space. Subjects who based their initial decisions regarding the sections only on perception would not be operating on the section itself but only on the solid. This would lead to errors in the identification of the section in many instances. All subjects in the sample were over 10 years of age and could be assumed to be either at the concrete or formal level of operations in their thinking (Flavell, 1963). They would therefore be likely to form a mental image of the solid and operate on that image, rather than base their decision on their perception of the solid alone.

There are at least two ways, consistent with Piaget's theory, as to how the section could be represented internally. The mental actions could have been the result of empirical observations of the situation together with perceptions of what had occurred, or they might have been logico-mathematical in nature, drawn from reflection on the actions of the tester and deduction from the properties of the solid

(Montangero, 1976). The transformation of this mental image to a drawing or to a set of distractors could also have been a result of either type of action. No attempt was made in the present study to determine in which fashion each subject was operating, or if he was operating in an entirely different fashion.

From the previous discussion it appears clear that the ability to section solids may depend on the child's previous experiences with geometric figures and objects. The direct experiences with the solids employed in this study consisted only of being able to observe the experimenter perform the cuts. Quite different results might be expected if the subjects themselves were to manipulate the solids.

The third major conclusion from Piaget's work on space and geometry is that notions of topological space develop first in the child and these notions are followed by the development of notions of projective and Euclidean space. Replications of Piaget's experiments confirm his findings as to the sequence of development of abilities to do particular tasks (Page, 1959; Lovell, 1959; Lovell, Healy and Rowland, 1962; Dodwell, 1963; Rivorie, 1961; Laurendeau and Pinard, 1970). More recent work by the Geneva group also supports the earlier results (Montangero, 1976). Criticisms of the sequence have centered more on the mathematical interpretation of the tasks, rather than on the tasks themselves (Lovell, 1959; Kapadia, 1974; Martin, 1976a, 1976b). Although the sequence of development is generally agreed upon, differences have arisen in other areas. The

age levels at which children experience success with particular tasks, such as the sectioning tasks, has varied from study to study. Also the age range for success appears to be considerably greater than that suggested by Piaget.

The act of sectioning solids may be described projectively as a "geometry of viewpoints", and in Euclidean terms as a "geometry of objects". The cutting of the solid involves movement of the cutting instrument and a consideration of the Euclidean properties of the solid such as length, angle size, and parallelism. The observer must consider the solid projectively from several viewpoints other than his own in order to envision the section as well as the entire solid. The final operation involves performing a Euclidean transformation of the section from the solid to a drawing. The ability to section solids can then be used as an indicator of the child's ability to operate in Euclidean space.

Van Hiele Theory

The Van Hiele theory of development in geometry was proposed in 1957 by P. M. and Tina Van Hiele. The discussion presented here is based on a summary of the Van Hiele theory by Wirszup (1976).

The Van Hieles proposed five levels in the development of geometric thought. They postulated that initially, young children perceive geometric figures in their totality. The shape or figure is viewed as a whole. Relationships among components of each figure and between figures are not

considered. However, at the second level these relationships begin to emerge in such a way that students recognize figures by their properties. The properties are not yet connected with each other. For instance, a student at this stage may know that a parallelogram is a quadrilateral with opposite sides equal and parallel, opposite angles equal, and diagonals which bisect each other, yet he does not know that the properties are connected with one another. This connection occurs at the third level when the properties are put into a logical structure with the possibility of one property following from another, however the role of axioms is not yet fully understood. Deduction appears only in conjunction with experimentation. At level four the significance of deduction emerges, followed at the fifth and final level by a Hilbertian standard of rigor when theory is developed without the necessity of concrete interpretation.

The Van Hieles noted discontinuities in the learning curve revealing the presence of these levels. Children often appear to be between levels when very little progress is made. Progress to a higher level proceeds under the influence of learning and depends on content and instruction. In Piaget's theory development from one stage to another may occur in the absence of direct instruction, but not in the absence of learning. The learning, however, occurs within an experiential context and not necessarily from direct instruction. Like Piaget's stages, however, the skipping of levels in the Van Hiele theory does not occur.

The Sectioning Experiments

The task of sectioning solid figures used in this study is based upon Chapter 9 of The Child's Conception of Space (Piaget and Inhelder, 1967). The following portion of this paper will discuss Piaget's results on the experiments concerning this task as well as the results from several other studies where the task of sectioning solids was under investigation.

Piaget

Piaget's method of reporting research in the form of dialogues and related discussions often makes it difficult to form generalizations. Piaget's subjects ranged in age from 4 to 12 years. Various solid shapes made of plasticine were used ranging in difficulty from the cylinder, prism, parallelepiped and hollow ball to the cone and eventually to more complex objects such as closed annular rings, a four pointed star, a cornet, a flex, and a helix. The exact cuts made on particular objects are difficult to ascertain from the dialogues given. Two methods of response were elicited from each subject. The child was asked to draw the expected surface and also to pick it out from a selection of comparison drawings. The children were given hints and suggestions in accordance with their ability. These hints ranged from beginning to make a cut to actually cutting a solid. This individual attention may well account for the early ages of success reported by Piaget.

The results of Piaget's sectioning experiments are

summarized by Holloway (1966). Prior to age four no meaningful responses were given to the sectioning tasks. Until age six the child was unable to show the section prior to the actual cutting. He had difficulty in distinguishing the internal viewpoint, which represents the section, from the visible form of the solid. The result was often a medley of viewpoints in a single drawing. At age eight the subjects experienced success on many of the sections and by age 12 Piaget claimed that the children were able to predict the sections of simple shapes including the cone with few errors. In Piaget's view, then, the child can effectively operate in Euclidean space by age 12.

Rivorie

Rivorie (1961) investigated the sequential development of representational space in children from 4 to 15 years of age. She concluded that the ability to section geometric solids was still not fully developed in all 14 year olds. In her opinion children could not operate at the Euclidean level before age 10 and many still had difficulty at age 14.

Dodwell

Several of Piaget's tasks were replicated by Dodwell (1963) with Canadian children, aged 5 to 11. Included in his tasks were six on sectioning solids. Dodwell reported stages similar to those of Piaget, but he found very little consistency from one cut or solid to another. Many of the older children in his sample still performed at an immature level on one or more of the tasks.

Lovell

Lovell (1965, 1971) generally concurred with Dodwell's results. He reported an increasing accuracy of the drawings with age, with the majority not very good before age 10 (Lovell, 1965). He also noted greater discrepancies between the stages reached by a child using various solids than Piaget admitted. He further noted that the advanced stages come rather later than those reported by Piaget (1971).

Boe

Boe (1966) administered sectioning tasks individually to subjects in grades 8, 10, and 12. She used four cuts on each of four solids, the cube, cylinder, rectangular prism, and the cone. Her methods of response were similar to those used by Piaget. The results were interpreted to be in disagreement with those of Piaget with Boe reporting that her subjects were not able to successfully section all solids and cuts. Her criterion for success was a perfect score. Males consistently performed better than females although differences were not significant. High ability students performed significantly better than average ability students who in turn performed better than low ability students. The cone was the most difficult of the solids with the oblique cut being more difficult than the other three cuts employed. No grade differences were detected although this was probably due to the advanced grade levels sampled.

Davis

Davis (1969), in an attempt to explain the variance between the results of Boe and Piaget, repeated Boe's sectioning experiment with subjects in grades 6, 8, and 10. He used basically the same solids and cuts as Boe but employed only the multiple choice mode of response. Groups of six students were tested simultaneously. Davis also included a short work period before the testing to acquaint subjects with similar tasks. It was concluded that grade 6 students scored significantly below those in grades 8 and 10 on every cut and every solid. This finding, when considered with Boe's results, suggests perhaps the ability to section solids stabilizes between grades 6 and 8. Davis also reported males scoring higher than females on all cuts and solids. High ability students scored higher than did low ability students. The cone and oblique cut again posed the most difficulty for the subjects. Davis interpreted his results to be in more agreement with those of Piaget than with those of Boe since he considered a perfect score not to be essential to full development of the ability to section solids.

Palow

Palow (1970) used an instrument consisting of photographs of solid figures with an arrow indicating the perspective from which the student was to view each solid. A multiple choice format was used for responses. The instrument was administered to classroom groups. Age was

found to be a significant factor with the ability to envision solids from different perspectives well developed by grade 6. Males and high ability students again were superior to females and low ability students respectively. No differences were detected among various socio-economic groups.

<u>Bober</u>

Two studies conducted at the University of Alberta have investigated the ability to section solids. Bober (1973) developed two forms of a sectioning test using the same four solids as used by Boe and Davis together with a square pyramid, a parallelepiped, a triangular prism, and a four pointed star. His subjects included students in grades 7, 8 and 9. He concluded that students at each of these grade levels had not attained the Euclidean level of thought. They failed to meet either Boe's criterion of a perfect score or the more liberal 75% criterion suggested by Davis. Grade 9 students were reported to score significantly higher on the test than either grade 7 or grade 8 students. No sex differences were detected in this study.

An experimental group was given a treatment rich in geometric experiences but not including the sectioning tasks used on the tests. This group scored significantly higher than did a control group on the sectioning posttest. Bober's results strongly suggest that the ability to section solids can be developed through appropriate geometric experiences.

Pothier

The study by Pothier (1975), also conducted at the University of Alberta, employed the same solids and cuts as those used by Davis and Boe. Her sample included students in grades 6, 8, and 10. Both methods of response were used after a short presession in which geometric figures and solids were discussed. Pothier reported scores similar to those of Davis yet significantly below a perfect score of 32. Males again scored higher than females. Grade 10 students scored significantly higher than grade 6 and 8 students. The oblique cuts and the cone again posed the most difficulty.

Chetverukhin

A study in the Soviet Union by Chetverukhin (1971) employed two unique sectioning tasks. In the first, a sphere located inside a cube, the diameter of the sphere being equal to the edge of the cube, was sectioned in four ways. The second involved the sectioning of a torus. Subjects were in grades 8, 9, and 10 (ages 16-19). The results of the first task indicated that a cut through the major axis of the cube was easier than a cut off the major axis but parallel to it, or than diagonal cuts. With the torus, a cut parallel to the major axis and tangent to the inner circle was more difficult than either a cut through the major axis or other cuts parallel to that axis. Males scored higher than females on both tasks.

Discussion of the Sectioning Experiments

The studies just reviewed had several differences in design. Different age levels of subjects were investigated. The method of testing differed in several respects, as did the activities of the subjects prior to the testing. Subjects were tested in individual situations in some studies and classroom groups in others. Solids and cuts were different in some of the studies. The studies, with the exception of Bober's, did not relate directly to the teaching of geometry.

Piaget and Inhelder (1967) contend that the ability to section solids is fully developed by the age of 12. The results of the other studies indicate that development continues until approximately age 14 and then levels off. Boe reported that even twelfth grade subjects still experienced some difficulties. The Van Hieles suggest that "discontinuities" exist between levels in the development of geometric ability. Perhaps a discontinuity exists in the ability to section solids at the 13 to 14 age range. If deductive ability is necessary to section solids, many subjects of this age may not yet have attained level three or level four on the Van Hiele scale.

The experiences of the students prior to and during the test may have accounted for some of the differences in the ages at which success occurred. For example, Davis reported higher scores at grade 8 than did Boe. Perhaps this was due to the short presession which Davis included in his experiment. The Van Hieles, unlike Piaget, claim that

instruction is necessary to move upward in the levels. Davis' presession, though short, could have provided that stimulus for many students. Piaget's dialogues could also be interpreted as instruction and thus success was attained at a lower age. Whatever the circumstance, the ability to section solids almost certainly increases with age.

Several of the studies reported significant differences between cuts and between solids. In particular, the cone presented difficulty on several cuts and the oblique cut was difficult for students of all ages. Boe reported a significant interaction between cuts and solids suggesting that particular cuts on particular solids created special difficulties. The transverse cut on the cone is an example of this. These results indicated that the ability to section solids may be a multifactor rather than a uni-factor ability. One purpose of this study was to test this conjecture.

Piaget did not categorize children by either sex or ability in his experiments. However, in almost every case where these variables were included in the design by other researchers, they accounted for a significant portion of the variance. It has been well documented that males out-perform females on most spatial tasks (Fennema, 1974; Mitchelmore, 1976). This pattern of results continues with the sectioning experiments. In every experiment where sex was used as a variable males scored as well as, or higher than, females on the sectioning tasks. Fennema (1974) suggested several reasons for this male superiority in spatial ability ranging from inherent mental factors to the kinds of toys children

play with and the play activities in which they engage.

In studies where the ability factor was studied, significant differences were found between high and low ability subjects. In addition, the high ability subjects outscored those of average ability who in turn scored higher than those of low ability. There seems to be little doubt that the ability level of a student is highly correlated with his success in sectioning solids.

The present study further examined these factors of grade level, sex, and ability level using modifications in the sectioning tasks and employing a group testing format.

Predicting Achievement in Geometry

As indicated by the studies reviewed in the previous sections of this chapter, considerable effort has been exerted on research concerning the sectioning of solids. These efforts have given us a great deal of information as to how children of a given sex, age level, IQ, and socio-economic status will reply to questions on sectioning solids. The studies revealed little, if any, information that is directly usable by a teacher in a classroom situation. The second major purpose of this study was to determine if the ability to section solids is in any way an effective predictor of success in the study of geometry. If this is the case, this information should prove valuable to the teacher of geometry.

Prior to 1960 very little geometry, even of an informal nature, was taught in the junior high and elementary

schools. The literature dealing with prediction of achievement in geometry therefore deals almost exclusively with senior high school students and with the traditional grade 10 Euclidean geometry course. The results of these studies are somewhat applicable to the present study since many of those concepts found in the high school geometry programs of previous courses now appear much earlier in the curriculum. No studies were located dealing with the use of sectioning solids as a predictor of geometry achievement.

Hanna (1966) provided the most recent summary of the literature on prediction of achievement in geometry. He reviewed the literature under the four headings: previously acquired abilities, aptitudes, interests, and temperament. The first two are of significance to the present study. All of the studies reviewed by Hanna in these categories were completed prior to 1950, a fact that Hanna indicated probably signifies a general acceptance of these earlier findings.

The results generally indicated the validity of past course marks and achievement tests in algebra, arithmetic and reading as predictors of achievement in geometry (at the high school level). IQ is also reported to correlate highly with geometry achievement. Hanna feels that little improvement can be made in prediction if we limit our efforts to the traditional domain. This study sought to make a contribution to that improvement by investigating the use of a test on sectioning solids as a predictor of geometry achievement.

Reviews of many of the same studies considered by

Hanna were conducted by Douglass (1935) and by Douglass and Kinney (1938). Their conclusions coincided with those of Hanna. In addition they suggested that achievement in geometry may be predicted with only a fair degree of accuracy when using any single predictor. The more useful predictions were obtained when two or more predictor variables were used. The best of these were prognostic test scores and marks received during the previous school year.

More recently Sowder (1974) examined the National Longitudinal Study of Mathematical Abilities (NLSMA) data for high school students and identified eight variables which when considered together resulted in a correct classification of over 90% of those students previously classified as high or low achievers in geometry. He noted that of these eight measures, four were prior achievement measures, hence lending support to the conclusion that past performance is a good indicator of present school success. Three of the remaining measures were labeled as cognitive scales. Two of them supposedly measured the ability to handle novel mathematical situations whereas the third, a paper folding test, tested ability to manipulate spatial patterns. The final predictor was an anxiety measure.

In the elementary school D'Augustine (1966) concluded that reading and arithmetic achievement were significant factors in achievement with topics of topology and geometry. Age was not found to be a significant factor. Williford (1972) concurred with these results.

A recent study by Guay and McDaniel (1977) examined the

relationship between mathematics achievement and spatial abilities among elementary school children. Spatial tests on serial integration, embedded figures, coordination of viewpoints, and surface development were used. They concluded that high mathematics achievers have greater spatial ability than low mathematics achievers. This relationship was maintained across grade levels and applied to both low and high level spatial abilities.

Previous achievement measures, particularly when used in conjunction with prognostic measures, appear to be the best predictors of achievement in geometry. In this study an attempt was made to determine if a test on sectioning solids will predict equally as well or can add to the accuracy of predictions of other variables.

Summary

What can be learned with respect to the four questions of this study from the above literature? First, much is known about the sectioning tasks. Age, sex, and cognitive ability are all important factors which influence the ability to section solids. The oblique cut is hypothesized to be more difficult than most other cuts on all solids. The transverse cut on the cone is also very difficult. Yet there is surprisingly little information on how students at various ages or grade levels function, particularly beyond age 12, and on whether sectioning is a uni-factor ability or constructed of several independent or related abilities.

The Van Hiele theory should enable one to describe the

empirical results from a more mathematical point of view. With respect to sectioning, Piagetian theory says that the ability to section solids is indicative of the child's ability to operate in projective space, that is, to consider viewpoints other than his own, and also in Euclidean space where he or she must consider Euclidean properties such as distance and angle size. Van Hiele theory allows for elaboration on Piaget's theory by considering the relationship between properties of the solids and of the sections and the methods by which these relationships are considered. Are the properties considered in isolation by the student or are they seen as following from one another? What is the role of deduction and must the child understand the deductive process to solve some sectioning tasks? The Van Hiele theory permits extension into these more mathematical arenas.

Of the research to date, only Bober (1973) has sought to relate sectioning and geometry instruction. While the prediction literature is limited and somewhat dated, it does suggest that the value of sectioning tests as predictors of achievement is open to question. To identify situations where the sectioning tests are of value as predictors of achievement was one purpose of this study.

CHAPTER III

The Sectioning Tests

The tests on sectioning solid figures were the central focus of this study. The first purpose was to determine how students at a particular grade level and of a particular sex and ability level responded to the sectioning tasks, and the abilities required for success on these tasks. Secondly, an attempt was made to determine the utility of one form of the sectioning test in predicting achievement in geometry. In the previous chapter, the results of studies using similar sectioning tests were reported. In this chapter the evolution of the present forms of the tests from previous studies and the ways they differ from the previous versions are described. Detailed descriptions are given of each solid and each cut used in the study, as well as for the protocols for administering and scoring the tests. Finally, the reliability of the sectioning tests is discussed.

Development of Tests

The task of sectioning geometric solids was first used by Piaget and Inhelder (1967) in their investigations into the child's conception of space. They describe their experiments as follows.

> The experiment consists simply of looking at objects made of plasticine, such as a cylinder, a prism, a parallelepiped or a cone, and predicting the shape of the surface produced when the solid is cut along various planes with a large knife. (p. 248)
>
> To ensure that the child's responses are a genuine product of his spatial or geometrical concepts, and not merely artifacts of the experimental technique, we invariably asked him to (a), draw the expected surface before the section is cut, and (b), pick it out from a selection of comparison drawings, both questions being put in the course of conversation aimed at following his train of thought. (p. 249)

From the descriptions and sample dialogues reported by Piaget and Inhelder (1967) it is difficult to determine from what position the child viewed the solid and the exact orientation of the cut. Only a few sample drawings were reported and very little indication was given as to what was included in the selections of comparison drawings. The dialogues used in the testing depended on the responses of the student and hence differed from student to student. These features are characteristic of Piaget's clinical method of experimentation.

Dodwell (1963), who used only drawing responses, developed a standardized dialogue for administering his sectioning tasks.

> The tester said "I am going to cut this roller (cylinder) in the middle like this (perpendicular to the main axis, indicated by a gesture). I would like you to draw the side you'll see where it has been cut... ." Then showing a cut section: "Did you think it would look like that?" (p. 147)

This dialogue was modified slightly for each task depending on the solid and cut involved. The dialogue was further modified in subsequent studies to include the multiple choice method of response, however changes were minor. Dodwell, like Piaget, did not provide exact descriptions of his cuts, or methods for determining what constituted a correct response.

The tasks were further refined by Boe (1966) who provided descriptions of each cut together with diagrams which indicated how each solid was held. She describes the cuts as follows.

> A longitudinal cut which was a perpendicular bisector through the major axis; a transverse cut which was perpendicular, but not a bisector, through the major axis; an oblique cut which traversed the solid figure oblique to the surface upon which it rested, beginning and ending within the bounds of the solid figure; and a parallel cut which was parallel to the surface upon which the solid rested. (p. 56)

These same four labels - longitudinal, transverse, oblique, and parallel - have also been used to describe the cuts used in studies by Davis (1969), Bober (1973), Pothier (1975), and Kuper (1975), as well as in the present study. The description of the cut was basically the same in each of these studies, however in some instances different sections resulted due to the solid itself being held differently. In this study care was taken to ensure that the orientation of each solid remained the same for all cuts, and that the direction of movement of the cutting instrument was the same

for a particular cut on all solids.

Piaget and Inhelder's (1967) discussion of their sectioning experiments was divided into three portions depending on the solids used. The first group of solids included the cylinder, prism, rectangular parallelepiped, and the hollow ball. The second portion dealt exclusively with the cone, while the last set included "complex objects" such as annular rings with both circular and square cross sections, a pair of circular discs connected by a parallelepiped, a four pointed star, a cornet, a flex, and a helix.

To introduce her subjects to sectioning tasks, Boe (1966) selected as sample solids, a sphere because of its familiarity, and a triangular prism because of its variety of sections. For her tests, the rectangular prism, cylinder, cube, and cone were selected. These same four solids were in turn used by Davis (1969) and Pothier (1975).

Bober (1973) used an oblique parallelepiped and a triangular prism together with the cube and cone for one form of the test, and added the square pyramid and four pointed star to the rectangular prism and cylinder for a second form. He reported the two forms of the test to be of equal difficulty on the basis of a pilot study, however his data, particularly that for grades 7 and 9, suggest that perhaps the second form was less difficult than the first. Bober also used Piaget's complex objects in his laboratory treatment. In the present study, the same two sets of solids used by Bober were employed.

Boe (1966) scored each drawing response either "1" for an acceptable answer or "0" for an inappropriate answer.

> An appropriate response implies in the case of straight line drawings, not more than one error or two symmetrical errors such as a rectangle for a square. In the case of curved drawings an appropriate response implied a clearly recognizable figure such as a circle or an ellipse. (p. 62)

This description, although helpful in scoring drawings as appropriate or inappropriate, does not provide precise criteria. Did angles have to be exactly 90° and did sides of a square need to be congruent? What level of tolerance was acceptable? The studies by Bober (1973) and Pothier (1975) also elicited drawing responses, however neither provided specific criteria for scoring the drawings. Bober indicated that a drawing was appropriate if "the figure was clearly recognizable as the appropriate figure". Pothier used the term "established expectation" but did not indicate what these expectations were. The present study developed specific criteria for scoring the drawings and they are presented later in this chapter.

Boe (1966) also developed selections of multiple choice distractors for each of the four cuts on the cube, rectangular prism, cylinder, and cone. The selections were based on pilot studies with children from grades 1, 4, 6, 8, and 10. These same sets of distractors were subsequently used by Davis (1969), Pothier (1975), and Bober (1973). Bober's experiment required an additional four solids for use in a pretest-posttest design. On the basis of his pilot work he

selected a triangular prism, a square pyramid, a four pointed star and an oblique parallelepiped as additional solids. Selections of distractors for the multiple choice questions dealing with these solids were also based on this pilot study.

Kuper (1975) made refinements in the drawings of several of the distractors for all eight solids. This was done to make the measurements of the distractors more proportional to those of the actual solids. Also, several of the distractors were changed as a result of her preliminary work with the instrument. Only minor alterations were made in Kuper's distractors for the present study.

The sectioning tests developed for this study were essentially the same as those used in the related works reported upon previously in this chapter and in Chapter II. Two forms of the sectioning test were developed and these were labelled Test I and Test II. Complete copies of these tests may be located in Appendix 1. The protocol for administering each test, the order of presentation of items on both the drawing and multiple choice portion of each test, the multiple choice distractors, and the correct answers are also included, as is a sample student answer booklet.

The activities prior to the administration of the sectioning tests have differed from study to study. Piaget and Inhelder (1967) did not use a presession, but presented their subjects with more information about the tasks early in the administration than was the case with later tasks. With some children they found it necessary to show one or

more sample sections. Boe (1966) introduced the tasks by demonstrating sample sections on the sphere and on a triangular prism. Davis (1969) used a 25 minute work period where the subjects cut up styrofoam objects and were asked questions about the cross sections. This could be construed as a training session. Bober (1973) used only the cuts on the sphere to introduce his tests. This exclusive use of the sphere may have given his subjects a false sense of security since all sections were the same and were no doubt "easy" in comparison to some of the other sections. Pothier's (1975) presession consisted of a short discussion on plane geometric shapes followed by a sample cut on the sphere. This was followed by a demonstration of the four cuts on the octahedron. In the present study, one cut on the sphere was illustrated followed by the four cuts on the octahedron. The sections on the octahedron were easy enough for the subjects to envision, or at least accept when they saw the section, and difficult enough to make them realize that they must apply themselves to the task and not rely on their perception of the solid alone.

Piaget and Inhelder (1967) and Boe (1966) both administered the tasks individually. Davis (1969) used small groups of six pupils. Bober (1973) and Pothier (1975) used the group format of testing. The present study used the class-group method of testing in an attempt to further test the results of the previous studies, especially those of Davis who tested in groups of six, and Boe who tested individually. Bober was more concerned with pretest-posttest

differences than with performance on individual items or on the test per se. Pothier's study, although similar in several aspects to the present study, included only grades 6, 8, and 10 and used only four solids. The present study provided additional information on the group method of testing as compared to the individual or small group method.

Description of the Solids

Each solid employed in the study was constructed of white cardboard with the exception of the sphere. The sphere was made from white styrofoam. The dimensions of each solid are presented below.

Sample Solids

a. Sphere: The radius of the sphere was 5 cm.

b. Octahedron: Each side of the regular octahedron was an equilateral triangle with sides 15 cm.

Test I Solids

a. Cube: Each side of the cube was a square with sides 15 cm.

b. Triangular Prism: The base of the prism was an equilateral triangle with sides 12 cm. The height of the prism was 20 cm.

c. Parallelepiped: Each of the six faces of any parallelepiped are parallelograms with opposite faces being both parallel and congruent. The faces of the parallelepiped employed in this study were 12 cm by 15 cm, 15 cm by 20 cm, and 12 cm by 20 cm. The dihedral angles between adjacent

sides measured 70^{o} and 110^{o}.

d. Cone: The cone had a circular base of radius 7.5 cm. The perpendicular height of the cone was 30 cm.

Test II Solids

a. Rectangular Prism: The rectangular prism was a parallelepiped with dihedral angles of 90^{o}. The faces were rectangles which measured 8 cm by 15 cm, 8 cm by 20 cm, and 15 cm by 20 cm.

b. Cylinder: The cylinder had a radius of 6 cm and its height was 20 cm.

c. Star: The face of the solid resembled a four pointed star. Each of the two faces was constructed by placing an equilateral triangle externally on each side of a square. Each side of the square was 10 cm. The two star shaped faces were 12 cm apart.

d. Square Pyramid: The base of the pyramid was a square with sides 15 cm. The perpendicular height was 20 cm.

For the purposes of tables throughout the remainder of this report the following abbreviations will be used to identify the solids: cube - S1, triangular prism - S2, parallelepiped - S3, cone - S4, rectangular prism - S5, cylinder - S6, four pointed star - S7, and square pyramid - S8.

Description of the Cuts

Each of the cuts employed in the study was determined by three factors: the orientation of the solid from the subject's point of view, the direction of movement of the cutting instrument, and the placement of the cut on the solid.

An examination of the cuts used in previous research (Boe, 1966; Davis, 1969; Bober, 1973; Pothier, 1975) revealed differences, both between and within studies, with respect to each of these factors. For example, Boe positioned the cylinder with the base parallel to the floor for the transverse cut and perpendicular to the floor for the other cuts. Davis positioned the base perpendicular to the floor for all cuts on the cylinder. Bober placed the cylinder in this position for the longitudinal and oblique cuts and Pothier for only the oblique cut. Similar illustrations may be made with respect to the direction of movement of the cutting instrument and the placement of the cut on the solid. In the present study, the orientation of each solid was kept constant for all cuts, and the direction of the movement of the cutting instrument remained the same for each cut on all solids. The placement of each cut of a particular type was consistent for all solids. A visual description of each cut on Test I and Test II, as seen by the tester, is presented in Figures 1 and 2 respectively.

The cube was held with one face parallel to the floor and one face pointing directly at the subjects. The

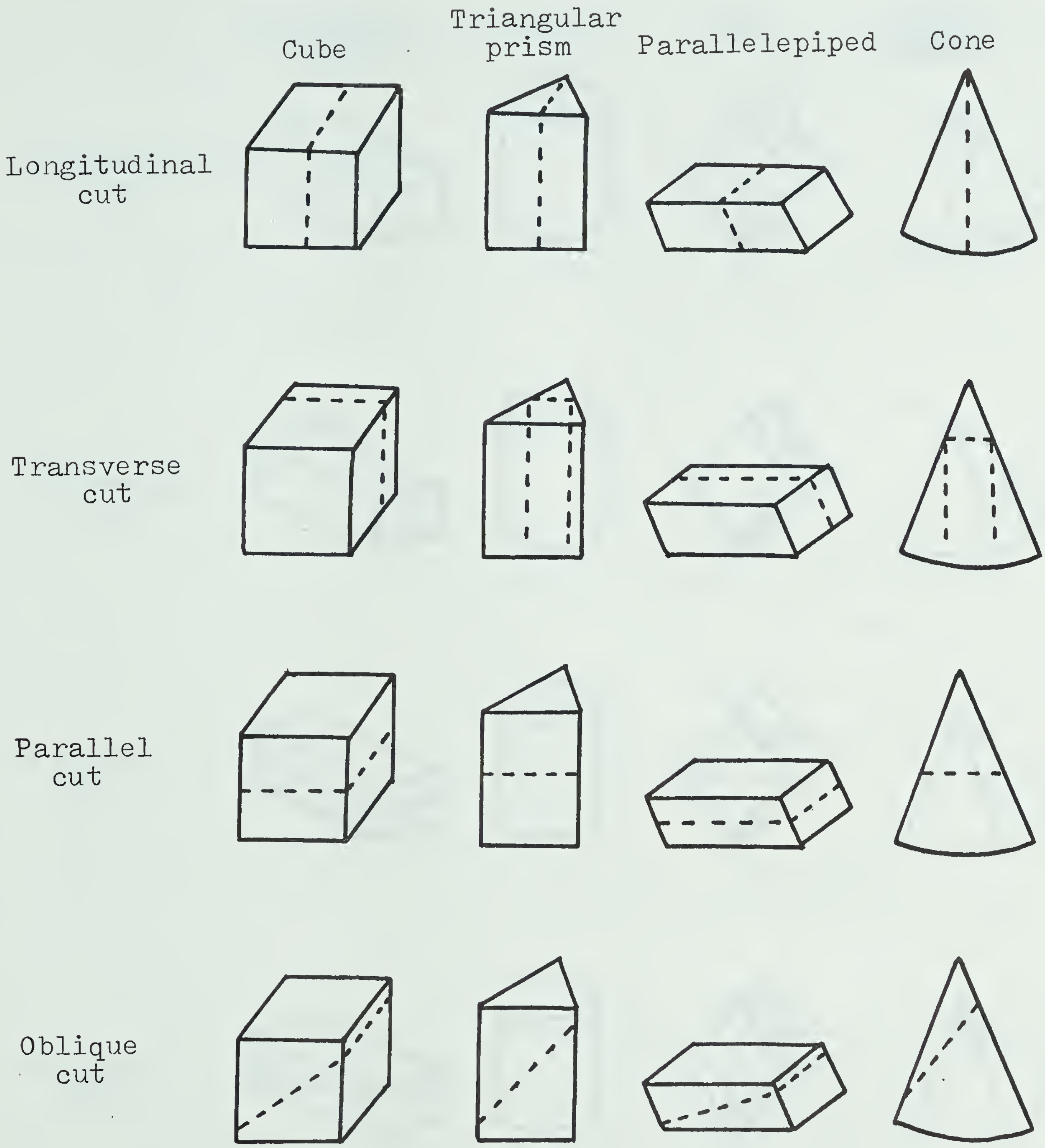


Figure 1

Test I Solids and Cuts

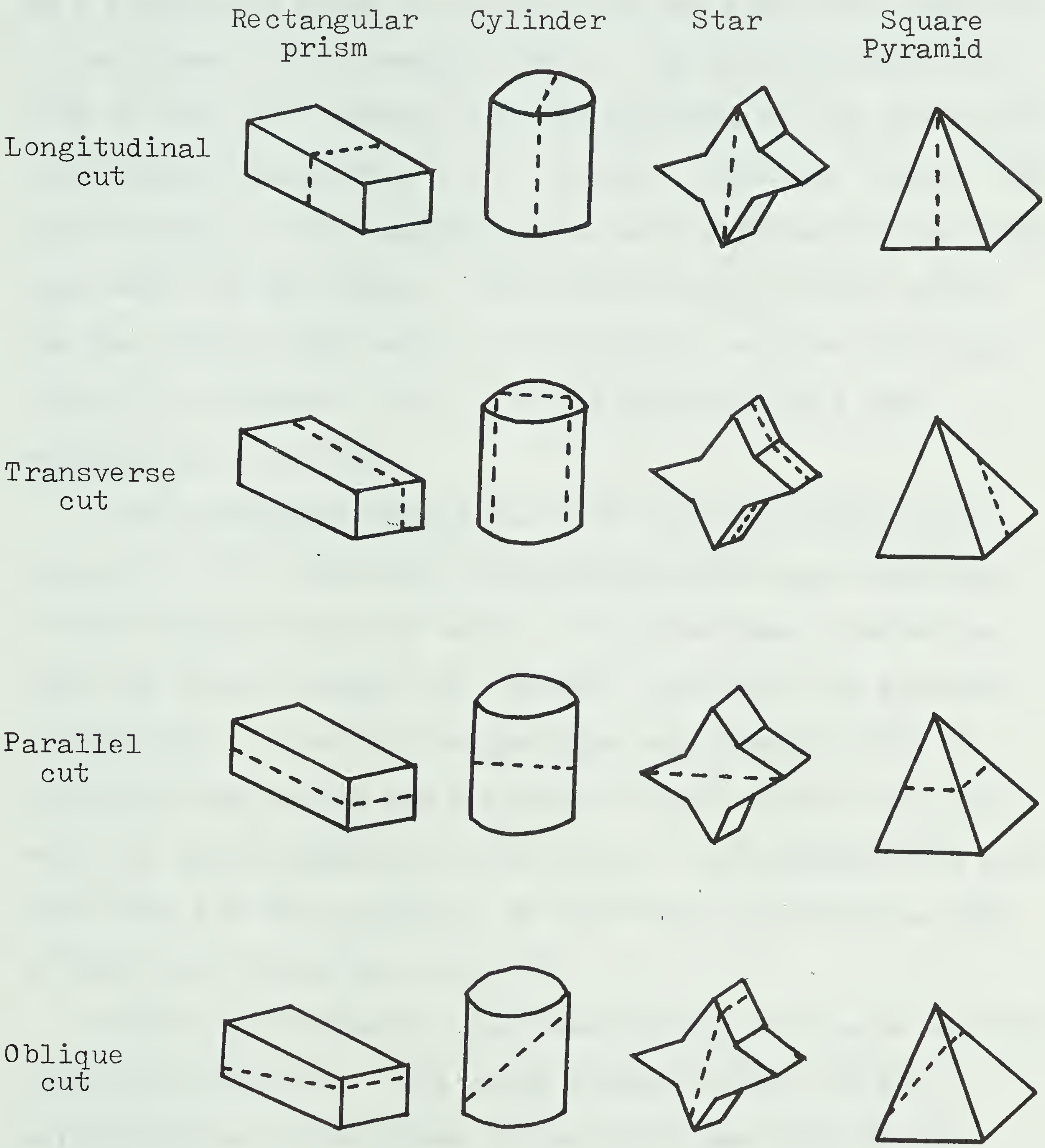


Figure 2

Test II Solids and Cuts

triangular prism had its triangular faces parallel to the floor and a vertical edge pointed directly at the subjects. As a result the plane of one face of the prism was parallel to the plane of the tester's body. The parallelepiped was held so that the largest face was parallel to the floor and the smallest faces were to the tester's left and right. The bottom edges of the smallest faces were further to the right than were the top edges. The face pointing at the tester had the bottom edge nearer to the tester and the top edge nearer the subjects. The cone was held with its base parallel to the floor.

The rectangular prism was held with the largest face parallel to the floor and the middle-sized faces pointing at the subjects and the tester. The star was oriented so that the plane through two opposite vertices was parallel to the floor. The star shaped face was perpendicular to the floor and facing the subjects. The cylinder was held with its faces parallel to the floor. The pyramid also was held with its base parallel to the floor and with one edge of the base facing the subjects.

The longitudinal cut was made through the major axis of the solid being cut. The plane formed by the cut was perpendicular to the plane of the floor and also to the plane of the tester's body.

The transverse cut resulted in a plane perpendicular to the floor and parallel to the tester's body. Each solid was cut approximately half way between the major axis and the front edge of the solid, the front edge being visible to

the subjects tested. The direction of movement of the cutting instrument for the longitudinal and transverse cuts was downwards.

The parallel cut produced a plane parallel to the floor. The intersection with the solid occurred midway between the uppermost and lowermost parts of the solid. The direction of the cut was made from the tester's right to his left.

The oblique cut formed a plane oblique to the plane of the floor, and perpendicular to the plane of the tester's body. The plane intersected the solid slightly below the top (or top edge) of the solid on the tester's right and slightly above the bottom edge of the solid on the tester's left.

Although the orientation of the solids, the direction of movement of the cutting instrument, and the placement of the cuts on the solids are described in detail above, the group method of testing employed in this study imposed limitations on the accuracy of these descriptions as they related to any individual student. It was not possible to hold the solids in such a manner that each student could view them from exactly the same perspective. Nor was it possible to illustrate a cut so that the plane formed by the cutting motion was always seen from the same viewpoint. To compensate for these factors each cut was illustrated a minimum of three times. The mechanics of this procedure are given in the next section.

In the tables of this report, the cuts will be identified by the following abbreviations: longitudinal - C1, transverse - C2, parallel - C3, and oblique - C4.

Protocols for Administering the Tests

The protocols for administering Test I and Test II are presented in Appendix 1 and are discussed only briefly here.

Prior to the administration of Test I, a short session was given to familiarize the subjects with the kinds of tasks they were to perform. A sphere was shown to the subjects and its properties were discussed. The subjects were then asked what plane shape would result if the sphere were cut into two parts. After some discussion, a model which could be taken apart was examined and it was shown that the answer was a circle. The tester then drew a circle freehand on the chalkboard and the subjects did likewise on their answer booklets. The entire procedure was repeated for each of the four cuts on the octahedron. Each of the plane figures was discussed, as well as what constituted a correct drawing. The criteria for correct drawings are given in the next section. Many subjects were concerned about their freehand drawing ability and this brief session helped to relieve them of this anxiety.

The session prior to the administration of Test II consisted only of a demonstration of a cut on the sphere and the longitudinal cut on the octahedron. This brief review allowed the subjects to recall the nature of the sectioning tasks from Test I. In cases where some subjects could not recall the nature of the tasks, additional cuts on the octahedron were demonstrated.

During each presession the direction of each of the four

types of cuts was illustrated without the use of a solid to make the students aware that the direction remained the same throughout the test. For example, the parallel cut was demonstrated by making a movement with the cutting instrument parallel to the floor. When the cuts were illustrated during the actual tests the names of the cuts were not used. Each cut was demonstrated by placing the cutting instrument on the solid and saying, "The knife goes in here" and moving the knife to the place of exit, without cutting through the solid, and concluding "and comes out here." In the case of the oblique cut the phrases "just below the top" and "just above the bottom" were used to further describe the entering and exiting positions. In instances where subjects asked for clarification, the cut was shown again and the direction explained in terms such as "straight up and down" or "parallel to the floor".

Prior to the test, the subjects were requested not to ask questions during the test. The only exception was to indicate by raising their arm that they wished a cut shown again. Despite this request there were several instances when questions were asked. The two most common of these were, "Which part do I draw?" and "Didn't you already show that one?" The first question was answered by asking the subject to recall what was done with the solids in the presession. The second was answered by telling the subject that the cut was different, however the resulting section might be the same, or it might also be different. These responses satisfied the questioners in all cases, at least in the sense that further questions were not asked.

It was important that for each sectioning task, each subject had the opportunity to view the cut from the desired direction. For this reason each task was demonstrated at least three times. The tester stood in front of the classroom near the center and demonstrated the cut while facing the back of the room. The same task was demonstrated a second time after the tester made a 45° turn to the right and a third time after a 45° turn to the left of the initial position. If an individual requested a further demonstration, the tester faced that individual and demonstrated the sectioning task. This procedure gave each subject an opportunity to view the solid from the desired direction.

A task was not repeated after it had been demonstrated and a new task considered. This meant that the subject had to decide on a response while the cut was being demonstrated and could not use information which he might obtain later in the test. In the same fashion once the multiple choice portion of the test was begun, subjects were not permitted to return to their drawing responses. Each of Tests I and II required between 40 and 60 minutes to administer.

The order of the items was different for the drawing and multiple choice portions of each of Test I and Test II. For each of the four subtests, the items were randomly selected in sets of four. Each set of four items included one for each solid used on that test. Since the answer booklet provided space for four drawings on each page, and each page of distractors contained sets for four items, the above procedure discouraged the students from comparing

their responses on a given solid.

Protocols for Scoring Drawing Responses

The method of response on Tests ID and IID was the drawing of a plane figure. After a sectioning task was demonstrated the subjects were asked to draw freehand the plane surface they would see if the solid were taken apart where it had been cut. Line segments which were straight, congruent, or parallel in the sections were to be represented as straight, congruent, or parallel in the drawings. Conversely, line segments which were curved, not congruent, or not parallel were to be represented in the drawings as curved, not congruent, or not parallel. Right angles and equal angles present in the sections were to be preserved in the drawings. The drawings were not expected to be in the same proportions as the sections. It was emphasized that the shape of the drawing was the critical factor and the orientation of the drawing on the paper had no significance as to its correctness.

Since the drawings were made freehand, they were not expected to be perfect and allowances were made for inaccuracies. The subjects were asked to be as neat and accurate as possible under the given conditions. In order to be as objective as possible in scoring the responses, criteria for marking the drawings were developed by the researcher. Although this allowed for objective scoring of the drawings, it was recognized that the selection of the criteria themselves tended to be quite arbitrary.

A sample of tests was randomly selected and the drawing responses were scored independently by the researcher and two colleagues using the criteria developed by the researcher. A total score on each test was determined by each rater and the scores ranked. Significant differences were found among the three sets of rankings using the Friedman test (Winer, 1971). Differences among the raters in scoring individual responses were detected for 15% of the drawings. These differences were discussed by the three raters and modifications were made in the criteria. More precise interpretations of the criteria were also discussed and agreed upon.

A second sample of tests was then selected and scored. The resulting rankings were again significantly different, with 7% of drawings being scored differently by the raters. The criteria were again discussed, interpretations clarified, and modifications made. The scoring of a third sample of tests resulted in high agreement among the raters on the rankings of total scores, and less than 3% difference in scoring drawings as correct or incorrect. This final set of criteria was then used by the researcher to score all remaining tests.

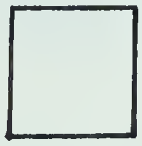
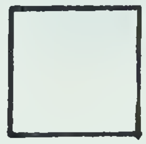
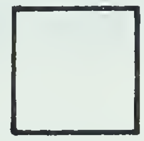
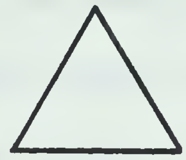
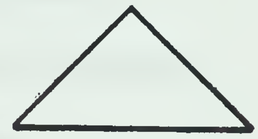
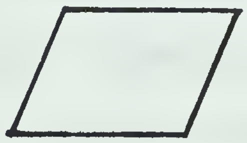
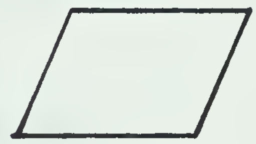
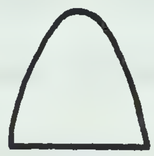
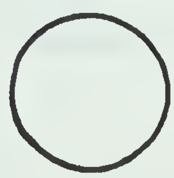
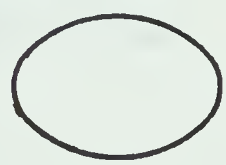
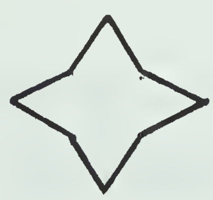
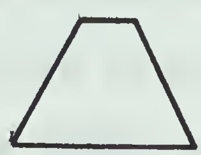
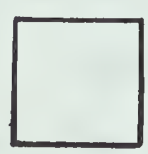
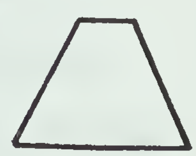
The correct plane figures representing each section are presented in Table 1. Following are the criteria used in scoring the drawings.

<u>Squares</u>

The longitudinal, transverse, and parallel cuts on

Table 1

Correct Plane Figure Representing Each Section

Cut Solid	Longitudinal	Transverse	Parallel	Oblique
Cube				
Triangular prism				
Parallelepiped				
Cone				
Rectangular prism				
Cylinder				
Star				
Pyramid				

the cube, and the parallel cut on the pyramid resulted in sections which were squares. To be classified as a square the drawing had to be a quadrilateral with the ratio of the longest side to the shortest side less than 1.2:1. Angles could not deviate more than 5^{o} from a right angle.

Rectangles

All cuts on the rectangular prism, the longitudinal and transverse cuts on the triangular prism and the cylinder, the oblique cut on the cube, and all cuts except the transverse cut on the star resulted in rectangles. To be classified as a rectangle the drawing had to be a quadrilateral with the ratio of the longest side to the shortest side greater than 1.2:1. The one exception to this was the oblique cut on the cube where the ratio 1.1:1 was accepted. Angles could not deviate more than 5^{o} from a right angle.

Equilateral triangle

The parallel cut on the triangular prism resulted in an equilateral triangle. The ratio of the longest side to the shortest side had to be less than 1.2:1.

Parallelograms

All cuts on the parallelepiped resulted in sections which were parallelograms. The figures had to be quadrilaterals with pairs of opposite sides having a difference in slope of not more than 10^{o}. Angles had to deviate more than 10^{o} from a right angle. The ratio of the

long side to the short side had to exceed 1.2:1 except for the longitudinal cut where 1.1:1 was acceptable.

Isosceles triangles

The oblique cut on the triangular prism and the longitudinal cut on the cone and the pyramid resulted in isosceles triangles as sections. In all three instances the ratio of the two "congruent" sides had to be less than 1.2:1. For the prism the ratio of the base to the longest "congruent" side had to be greater than 1.2:1 while on the other two solids the ratio of the shortest "congruent" side to the base had to be greater than 1.2:1.

The vertical angle of the isosceles triangle formed by the oblique cut on the prism was approximately 90°. Because of the orientation of the prism and the direction of the oblique cut, it was not possible for the vertical angle resulting from any oblique cut on this solid to be 60° or less. Hence, only angles in the range 65° to 110° were considered acceptable. For the other two cuts it was required that the vertical angle be less than 60° and the base angles less than 80°.

One Nappe of a Hyperbola

The transverse cut on the cone resulted in the section being one nappe of a hyperbola on a flat base. To be correct the drawing had to have a flat base and the sides had to be curved. The top also had to be curved and not peaked.

Circles

Both the parallel cut on the cone and on the cylinder resulted in a circle as the section. To be considered a circle the drawing had to resemble a circle with no two diameters having a ratio greater than 1.2:1.

Ellipses

Both the cone and cylinder yielded ellipses as sections when cut obliquely. The ratio of the major axis to the minor axis had to be greater than 1.2:1. Both sides had to be curved with the curvature being similar at each end.

Star

The transverse cut on the star resulted in a four pointed star as the section. The four "points" of the star were equilateral triangles erected externally on a square base. The ratio of the longest to the shortest side of each triangle could not exceed 1.2:1. The base of each triangle could not differ by more than 20% from any other base.

Isosceles trapezoids

The transverse and oblique cuts on the pyramid resulted in sections which were isosceles trapezoids. The drawing had to be a quadrilateral with base angles less than 80° and not different by more than 10°. The ratio of the "congruent" sides could not be greater than 1.2:1.

Appendix 2 contains illustrated examples of common

errors made on drawing responses.

Reliability of the Sectioning Tests

None of the previous reports on studies concerned with sectioning solids included a discussion of the reliability of the sectioning tests. Kuder-Richardson 20 coefficients were calculated for each of the two tests used in this study. The coefficients were .81 for Test I and .91 for Test II, indicating that each test was internally consistent. The correlation between Test I and Test II was .76.

Piaget and Inhelder (1967) claimed that the drawing and multiple choice methods of response are equivalent means of testing the pupil's ability to section solids. Boe (1966) investigated this claim and found a correlation of .55 between all 1152 responses via the drawing method and the corresponding multiple choice responses. She concluded that this coefficient was not high enough to justify Piaget's claim.

In the present study correlations were calculated between the drawing and multiple choice responses for each sectioning task. These correlations are located in the diagonal of Table 41 for items on Test I, and Table 44 for Test II. Both tables can be found in Appendix 3. The correlations range from .01 to .55 with a mean of .30. These low correlations support Boe's contention that the two methods of response do not measure the same thing.

The correlation coefficient between total scores on Tests ID and IMC was found to be .69 and between Tests IID

and IIMC, .81. When the scores on Tests ID and IID were added, the correlation between this total score and the overall total score of Tests IMC and IIMC was .85. These correlations suggest that for a set of sectioning tasks, the drawing and multiple choice modes of response are comparable means of testing sectioning ability. This interpretation supports Piaget's claim and is contrary to that made by Boe.

An item analysis was performed on each of Tests I and II. The difficulty, reliability, discriminating power, and biserial correlation of each item with the test were calculated. These results are reported in Tables 45 and 46 and can be found in Appendix 3.

Summary

In the first part of this chapter the sectioning tests were discussed as they had been used in previous research studies. Based on this previous work, changes were made in the instruments for use in this study. Solids and cuts were described in detail as were the protocols for administering and scoring the tests. The reliabilities of the tests were discussed briefly and evidence presented indicating that the sectioning tests were internally consistent. However, it was pointed out that the drawing and multiple choice methods of response may measure different aspects of spatial knowledge.

In the next chapter the sample and design of the study are described. The tests discussed in this chapter occupy a central position in the design of the experiment.

CHAPTER IV

The Experimental Design

The purposes of this study were to provide an indepth analysis of the ability to section geometric solids, and to investigate the relationship between this ability and achievement in geometry in existing classrooms. A detailed description of the sectioning tests, together with how they were administered, was presented in the previous chapter. In the present chapter the design of the experiment is described as it relates to each of the two purposes. The questions and related hypotheses to which answers were sought and the methods used to analyze the data are also presented.

Basic Design

Throughout the study, an effort was made not to disturb the existing ecology of the participating classrooms. To ensure the utility of the sectioning test in predicting geometry achievement for the classroom teacher, classrooms were investigated as they existed. The regular mathematics teachers taught their scheduled geometry units and used the methods of instruction and evaluation that they normally used. The only restriction was that the teachers were asked not to include any material on sectioning solids in the unit. A more detailed discussion of the content and methods used in selected classrooms is presented later in this chapter.

The researcher administered Test I to each class participating in the study just prior to their regular unit on geometry. The time of administration varied from early November until mid-May due to differences in times when geometry was taught at different grade levels and in different classrooms. The test was given during the regular mathematics period and required from 40 to 60 minutes to complete. The younger students generally required more time than did the older ones. The mathematics teacher remained in the classroom in most cases, but did not participate in the administration of the test.

The geometry units varied in length from 4 to 7 weeks depending on the grade and material covered. Teachers were asked to maintain a log, briefly describing the content and methods used, and to include copies of any tests given. The instructions given to participating teachers are included in Appendix 4.

Immediately following the unit on geometry, the researcher administered Test II. The procedure followed paralleled that used for Test I. Subjects were not given their results from Test I until after Test II was completed.

The above procedure permitted both purposes of the study to be carried out. For the purpose of analyzing the ability to section geometric solids, a stratified random sample of 432 subjects was selected from the population of students tested. A detailed explanation of this sampling procedure is given in the next section. For the purpose of investigating the relationship between sectioning ability

and geometry achievement, two situations were identified at each grade level tested. Each of these situations consisted of all subjects enrolled in one or more of the classes tested. They were described in detail following the description of the stratified sample.

Sample

Two to three classes from each of grades 5 through 10 were requested from each of the Edmonton Public and Edmonton Separate School systems. It was requested that the schools be as representative as possible of the systems at each grade level. The classes included those previously obtained for use in studies by Ong (1976) and Kuper (1975).

The researcher was assigned to five schools from the Public system and eight schools from the Separate system. Nineteen classes from each system were involved. A list of schools and the grades of participating classes are presented in Appendix 4. A list of participating teachers is also included.

Overall 1,004 subjects were tested. Of these 155 were absent for either Test I or Test II. Lorge-Thorndike IQ scores were collected from cumulative records and were used as a measure of ability. Scores resulting from tests administered within the 3 years prior to this study were available for all but 94 subjects. This resulted in full data for 755 subjects.

The Public school system reports IQ scores as a total raw score, whereas the Separate system reports both verbal and non-verbal scores as percentile ranks. The total raw

scores were converted to percentile ranks based on a mean of 100 and standard deviation of 16. The verbal and non-verbal percentile ranks were converted to standard scores, averaged, and this average converted to a single percentile rank. Three ability groupings were used in the analysis and labeled as high, average, and low. To ensure an accurate classification according to ability, subjects whose IQ fell between the 42.5 and 47.5 percentiles and between the 67.5 and 72.5 percentiles were eliminated from further analysis of the test. This resulted in 85 subjects being eliminated and left a sample of 670. Approximately one-third of these were classified as being each of low, average, or high ability.

Grade level and sex, in addition to ability level, were employed as independent variables. This resulted in a 6 x 2 x 3 factorial design with 36 cells. The number of subjects in each cell ranged from 12 to 30. In order to maintain equal frequencies in the cells, subjects were randomly eliminated resulting in 12 subjects per cell and a final sample size of 432. This sample was used in the analysis related to the first purpose of the study.

To determine the utility of Test I in predicting achievement in geometry, single classes or groups of classes were selected from among those tested. In cases where groups of classes were selected, each class in the group studied the same content under similar conditions. Two situations were chosen at each grade level. The classes not chosen contained too few students for a meaningful analysis. Each situation is described briefly below. The descriptions

are based on information provided by the teachers involved.

Situation 5-1

This grade 5 class was located in a Separate school in northeast Edmonton. The class was homogeneous in ability and complete data were available on 30 of 35 students in the class.

The unit in geometry followed a unit on estimation. It was 4 weeks in duration and included material from Chapters 5 and 12 of Elementary School Mathematics - Book 5 (Eicholz and O'Daffer, 1969a). The topics studied included the use of the compass and protractor, angle measure, perimeter and area of simple plane figures, and volume and area of cuboids. Some work with constructions was also completed.

The teaching method was primarily one of demonstration by the teacher followed with practice by the students. Models constructed of paper and plastic were used to illustrate some concepts. The evaluation of the unit consisted of two short tests, one given at the end of each chapter. The tests were those provided to accompany the textbook. The unit grade was arrived at by averaging the two test scores.

Situation 5-2

This group consisted of all three grade 5 classes in a Public school in west Edmonton. The grade 5 classes were taught by a team of three teachers who planned their activities jointly. Of 71 students in the three classes, complete data were available on 63. Fifteen of these were used in the analysis to cross-validate the prediction equations.

The previous unit of study was on rational numbers and included the addition and subtraction of fractions and

decimals. The geometry unit was 4 weeks in duration and included the same content as that described in situation 5-1. In addition to this content, Chapter 14 of <u>Elementary School Mathematics - Book 5</u> was also studied. This included introductory work in coordinate geometry where the students learned to identify the coordinates of points in the Cartesian plane.

Again the teaching approach emphasized giving the students practice in the concepts they were to learn. More attention was placed on the nomenclature of the point-set approach to geometry than in the previous situation. Evaluation consisted of a teacher-made test given at the conclusion of the unit. Items were similar to those practiced during the unit.

<u>Situation 6-1</u>

This class was from the same school as the grade 5 class in situation 5-1. Complete data were available on all 33 students in the class.

The unit in geometry followed a unit on elementary number theory. The unit was 5 weeks long and included material from Chapters 5 and 11 in <u>Elementary School Mathematics - Book 6</u> (Eicholz and O'Daffer, 1969b). The concepts in Chapter 5 were extensions of those studied the previous year, including the terminology of the point-set approach to geometry, and fundamental work with construction, congruence, areas, and perimeter. Chapter 11 contained introductory work on the circle and extensions of previous

work on volume and space geometry.

The teacher described his approach as lecture and discussion. The students' work closely followed the exercises in the textbook. The only instance of work with concrete materials was the construction of the cube, tetrahedron, and octahedron. The final unit grade was arrived at by averaging the results on the two chapter tests which accompanied the textbook series.

<u>Situation 6-2</u>

The two grade 6 classes described below were located in a Public school in northwest Edmonton. There were 59 students in the two classes for which there were complete data on 56. Fourteen of these students were used for cross-validation purposes.

Both content and teaching format were very similar to that described in situation 6-1. The previous unit was on decimals. The same chapters in the same textbook were used in both situations. In this situation the emphasis was on the terminology used in geometry. Assignments were given regularly and the students received considerable practice on the concepts of area and volume. The evaluation was a teacher constructed test given at the conclusion of the unit.

<u>Situation 7-1</u>

Situation 7-1 consisted of two grade 7 classes in a large Public junior high school located in south Edmonton. Both classes were taught by the same

instructor. There were 58 students in the two classes and complete data were obtained on 54. Thirteen students were used for cross-validation of the prediction equations.

The geometry unit was 4 weeks in length and was preceded by a short unit on measurement. The content consisted of the grade 7 geometry unit in the Junior High Mathematics Consortium (1975). This program is objectives-based and the geometry unit for grade 7 contained 24 objectives. The objectives included work on congruence of polygons; slides, flips and turns, and their basic properties; classification of polygons; and parts of the circle.

The teaching method was mostly demonstration on the overhead projector for the motions and the use of the discovery approach for work with polygons. The students discovered many of the properties of polygons through the use of the three basic motions.

The final test consisted of 35 multiple choice items together with some questions on performing motions. The multiple choice items were from those provided by the Edmonton Public School system.

Situation 7-2

The classes involved in this situation were also involved in a study proposed by Kuper (1975). These grade 7 students were located in three schools from the Edmonton Separate system. All were located on the west side of Edmonton. There was one class in each

school which received Kuper's treatment. Data were available on 61 of 74 students in these classes. Fifteen of the students were used for cross-validation purposes.

The treatment consisted of the unit Making Rectangular Solids (Kuper and Walter, 1975) and required 10 to 12 class periods to complete. The lessons were given in mathematics-option classes over a 10 week period. The unit consisted of a booklet of activities which the students carried out and discussed with their teacher. The students constructed boxes of various sizes given the possible lengths of the sides. They then discussed which boxes had the greatest volume and later calculated that volume. Work was then done on folding nets of squares into open boxes. The students discovered which nets folded into boxes and which did not. Problems were then posed about the diagonals of the cubes and experimentation done by the students.

The teaching method used in the unit consisted mainly of letting the students construct objects and discover the properties themselves. The teacher's role was to assist the students when necessary and to lead discussion at various points in the treatment. Kuper devised a test for the unit consisting of several problems which required the students to determine if a given net could be folded into a particular solid. Several other spatial type problems were also included.

Situation 8-1

The two classes in this situation were in the same school as those in situation 7-1. The classes were taught by different teachers who planned jointly and used the same evaluation instruments. Of 55 students in the two classes, complete data were obtained on 49. Twelve of these were used for cross-validation of the prediction equations.

The unit was 7 weeks in duration and included the objectives from the grade 8 geometry unit in the Junior High Mathematics Consortium (1975). It followed a unit on measurement. Objectives in the geometry unit included the study of the properties of slides, turns, reflections, and slide reflections and the use of these motions in the study of angles, triangles, and other polygons. The teaching method emphasized the discovery of the properties of the various motions and geometric figures. Frequent use was made of dot paper and graph paper. Teacher-led discussions were used frequently to arrive at generalizations. The students spent a large proportion of their time working on exercises related to the motions.

Evaluation of the unit consisted of two multiple choice tests administered at the end of the unit. The tests contained a total of 65 items and were obtained from the item bank maintained by the school system.

Situation 8-2

This situation consisted of the two eighth grade classes used by Ong (1976). Both classes were in a Separate school in north central Edmonton and were taught by the same teacher. One class received the regular geometry program while the second class received a special treatment of the same objectives.

The geometry unit followed a short unit on measurement. The objectives of the 6 week unit were identical to those described for the program in situation 8-1. In both classes in this situation the development of the concepts followed that suggested in the Junior High Mathematics Consortium (1975). The difference in the treatments received by the two classes was that the first group received only the exercises contained in the program. These exercises were convergent in design, usually requiring a single correct solution. The second group received inventive divergent questions requiring a variety of responses, in addition to the exercises received by the first group. This inventive treatment thus required the teacher to elicit inventive divergent solutions from the students. This was not done in the regular group.

Four tests were given at the conclusion of the unit. These included an achievement test on the unit, a test on area, a creative motion geometry test and a creative geometry test. Each of these instruments as well as the two treatments is described in detail by Ong (1976).

Situation 9-1

These two grade 9 classes were in the same school as the classes in situations 7-1 and 8-1. They were both taught by the same teacher. Data were available on 52 of the 56 students in the two classes. Thirteen of these were chosen for cross-validation purposes.

The geometry unit followed a unit on rational numbers and was 6 weeks in length. The unit consisted of the objectives in the geometry portion of the grade 9 Junior High Mathematics Consortium (1975). Some supplementary laboratory activities were selected from Geometry (Armour, 1974). The unit contained a review of area and perimeter concepts followed by several objectives dealing with Pythagorus' theorem. The major portion of the unit was work on surface areas and volumes of a variety of three dimensional objects. The teaching method was described by the teacher as lecture followed by seat work. Physical models were used for demonstration purposes as often as possible. The students spent a large portion of time calculating surface areas and volumes by substituting into formulae. Evaluation of the unit was done by a 36 item multiple choice test given after the unit had been completed.

Situation 9-2

All three grade 9 classes in a Separate school in west Edmonton were used in this situation. All three classes were instructed by the same teacher. Compete data were

available on 86 of 93 students of which 21 were used to cross-validate the prediction equations.

The unit was 7 weeks in length and included the same objectives as those in the previous situation. More emphasis was placed on reviewing the work from previous grades than was the case in situation 9-1. This teacher made ample use of models, the overhead projector, mobiles and charts in her teaching. She adhered closely to the lesson development suggested in the learning package of the Junior High Mathematics Consortium (1975). For the most part the concepts were demonstrated by the teacher and exercises completed by the students. Five multiple choice tests were administered at regular intervals over the 7 week period and the unit score was the average of these test scores.

Situation 10-1

This was a Mathematics 10 class in a large Public high school in south Edmonton. Mathematics 10 is a course recommended for the top 40 to 60% of students in the tenth grade. There were 37 students enrolled in the class and data were available on 34 of these.

The geometry unit consisted of the first seven chapters in Geometry   A Modern Approach (Wilcox, 1968). The material was covered in one 80 minute period per day over a 7 week period. The first four chapters contained basic terminology and concepts related to sets, relations, postulates, and proof, much of which was review of previous work. The emphasis in the unit was on the latter three

chapters which dealt with deductive geometric proofs using theorems related to congruent triangles and parallel lines.

The teaching approach employed was lecture and demonstration. The majority of the students' class time was spent on doing exercises and proofs. The teacher discussed the proofs with students both individually and as a class. One test was given after the first four chapters and a final examination given on the entire unit. The final examination was one-half multiple choice questions and one-half proofs. It was constructed by the regular teacher.

Situation 10-2

This group was a Mathematics 13 class in the same school as the class in the previous situation. Mathematics 13 is offered to the middle 25% to 35% of the students in grade 10 and leads to university, technical institutes or apprenticeship programs. Of 32 students in the class, 29 completed the unit.

The unit lasted 4 weeks and contained the material in Chapter 2 of Principles of Mathematics 1 (Dean, Graham and Moore, 1970). The content included classification of angles, triangles, and polygons; necessary and sufficient conditions for congruent and similar triangles; Pythagorus' theorem; parallel lines; and areas of polygonal regions. The approach to these topics was primarily inductive rather than deductive in nature. The teaching method was mainly discussion of the concepts with the students working on the exercises in the textbook. Filmstrips were used to

illustrate some concepts.

Short quizzes were given regularly and used directly as part of the lessons. The final test contained numerous items on terminology, others on numerical applications of the geometric concepts, and several items on the recognition of congruent triangles.

Questions and Hypotheses

The two purposes of this study were fulfilled by examining four research questions. This section of the report restates these questions and the related hypotheses are presented. The statistical procedures used to test these hypotheses, and thus provide answers to the questions, are also outlined.

Question 1

How do students respond to sectioning tasks involving particular cuts and solids?

This question was answered by first considering the proportion of students at each grade level who answered each sectioning item correctly. Piaget (1964) has used the criterion that a problem is considered to be solved by children of a certain age when 75% of the children of this age respond correctly. In this study a more conservative criterion of 70% was used due to the group method of testing.

Boe (1966) reported a significant interaction between cuts and solids in her experiment. Although neither Davis (1969) nor Pothier (1975) formally tested for interactions of this type, their data supported Boe's results. To

examine further the relationship between cuts and solids the following hypothesis was tested for Tests ID, IMC, IID and IIMC by employing a chi-square test of independence.

> Hypothesis 1: The ability to identify the section resulting from a given cut on a solid is independent of that solid.

Finally, a comparison was made between the performances on sections involving the cube, cone, rectangular prism, and cylinder obtained in this study with those obtained by Boe (1966), Davis (1969), and Pothier (1975). This comparison was descriptive in nature and served to give an indication of the consistency of the results among the different sectioning experiments.

<u>Question 2</u>

Are there differences in the ability of students of a particular sex, grade level, and ability to section solids?

The previous studies of Boe (1966), Davis (1969), Bober (1973), and Pothier (1975) all examined one or more aspects of this question. In each instance an analysis of variance (ANOVA) was used on each of the drawing and multiple choice subtests separately, or on the subscores for each solid or item. ANOVA fails to take into consideration the correlations between the various subtests in a multivariate experiment. On the other hand multivariate analysis of variance (MANOVA) makes use of these correlations and should be employed in cases where the dependent variables are correlated, as was the case in the present study (Tatsuoka, 1971; Bay, 1969a; Morrison, 1967). Tests were administered on two separate occasions. Each included

a drawing and a multiple choice subtest. Each of these four tests, ID, IMC, IID, and IIMC were considered as a dependent variable for purposes of testing the following hypotheses.

> Hypothesis 2a: There is no significant difference between the grade mean vectors on Tests ID, IMC, IID, and IIMC.
>
> Hypothesis 2b: There is no significant difference between the sex mean vectors on Tests ID, IMC, IID, and IIMC.
>
> Hypothesis 2c: There is no significant difference between the ability mean vectors on Tests ID, IMC, IID, and IIMC.

Prior to arriving at a decision as to whether to accept or reject these hypotheses, possible interactions between the three main effects of grade level, sex, and ability were examined. In those cases where the main effects were interpretable and significant, an attempt was made to determine the exact location of the significant differences.

When hypotheses such as those above are rejected in a multivariate experiment, the differences may be further examined by the use of individual univariate tests on each dependent variable (Amick and Crittendon, 1975). This combination procedure of MANOVA followed by individual ANOVA's results in a consistent experiment-wise error rate and keeps the probability of a group of errors occurring low (Hummel and Sligo, 1971). A three way ANOVA was completed on each of the four dependent variables used above to further examine the hypotheses of no significant differences due to either grade level, sex, or ability on each of the sectioning tests.

Question 3

Is the ability to section solids a uni-factor trait or is it a composite of several independent or related abilities?

This question was investigated by considering each item on each of Tests I and II as a variable and determining the correlation matrix for each of these two sets of variables. Principal axis factor analysis was carried out on each of these correlation matrices using squared multiple correlations in the diagonal as estimates of the communalities. The factors extracted were rotated by the varimax procedure to approximate orthogonal simple structure. Since the investigation was exploratory in nature and the intent was to determine underlying traits affecting sectioning ability, the method of principal axis factoring was selected rather than principal components analysis (Korth, 1975). Based on observations of the factor structures obtained, Hypothesis 3 was tested for each of the two sets of variables.

> Hypothesis 3: The ability to section solids is a uni-factor trait.

In the event that the ability to section solids was not a uni-factor trait and Hypothesis 3 was rejected, several factors have been suggested by the literature reviewed earlier in this report. Possible factors included one determined by the method of response, particularly the drawing response. Other possibilities included factors which resulted from a particular solid or cut. An oblique factor was a distinct possibility. It was also possible that a given cut on a particular solid required a unique ability.

In addition to the above possibilities, factor structures might be different for subjects at different grade levels, of a different sex, or of different ability levels. Each of these possibilities was explored and reported upon by observing the factor structures.

Question 4

Is the test on sectioning solids useful to the classroom teacher for predicting achievement on geometry?

For each of the 12 situations described previously, correlations were found between Tests ID, IMC, and I and the achievement score on the geometry unit. In those instances where each of these correlations was not significant, it was concluded that the sectioning tests were of limited use to the teacher of geometry and further analysis was not pursued. In situations where the correlations were significant, additional correlations were examined between the geometry achievement score and other predictors which were readily available to the teacher. These included sex, age, previous year's final mathematics grade, the achievement score on the previous mathematics unit, and the IQ score. For some situations only the total IQ score was available whereas for others both verbal and non-verbal scores were available. In each instance the score recorded in the cumulative record was used in the analysis. If one of the sectioning tests correlated higher than any other available predictor, then Hypothesis 4 was rejected and it was concluded that the sectioning test was of possible use to the teacher. In all other cases the

technique of stepwise regression (Draper and Smith, 1966) was used to test Hypothesis 4.

> Hypothesis 4: The efficiency of prediction of geometry performance is not significantly improved by adding tests on sectioning solids to a battery of other student scores including age, sex, the previous year's final mathematics grade, the last mathematics test score and IQ.

In addition to the above, Hypothesis 4a was tested to determine if the sectioning tests significantly imporved the prediction equation obtained by using only the previous year's final mathematics grade <u>or</u> the last mathematics test score <u>or</u> the IQ score. In instances where one of the sectioning tests correlated more highly with geometry achievement than did the individual predictor, Hypothesis 4a was also rejected.

> Hypothesis 4a: The efficiency of prediction of geometry performance is not significantly improved by adding tests on sectioning solids to the previous year's final mathematics grade <u>or</u> the last mathematics test score <u>or</u> the IQ score.

In all cases where two or more classes were available in a situation, approximately 25% of the sample was removed for purposes of cross-validation of the prediction equation (Tatsuoka, 1969). The predicted scores were calculated and the correlation between these predicted scores and the actual scores was determined. This cross-validation multiple-R was compared with the original multiple-R corrected for shrinkage as suggested by Tatsuoka (1969).

Analysis of Data

The analysis of the data was carried out on the University of Alberta computer using the programs provided by the Division of Educational Research at the university. Programs used included TESTØ4 for the item analyses, DESTØ2 for correlations among tests, MULV35 for the univariate analyses of variance. FACT18 was used for the principal axis factoring and MULRØ6 for the stepwise regression portion of the analysis.

The results of these analyses are presented in detail in the next chapter and discussed in Chapter VI.

CHAPTER V

Results of the Study

The results of the study are presented in this chapter which contains four main sections corresponding to the four main questions posed in Chapter I and restated in the previous chapter. The first three questions related to the first purpose of the study - to provide an indepth analysis of the ability to section geometric solids. The fourth question related to the second purpose - to investigate the relationship between the ability to section solids and achievement in geometry. The results presented on the following pages will be further discussed in the concluding chapter of this report.

Question 1

How do students respond to sectioning tasks involving particular cuts and solids?

This question was answered in a descriptive manner by observing the proportion of students answering each sectioning item correctly. The percentage of students at each grade level giving a correct response to each item on Test I is presented in Table 2. An item was considered to be solved by children at a given grade level when 70% of the students at that grade level could answer the item correctly. The percentage of students who correctly answered each item tended to increase as the grade level increased. Thus, if

Table 2

Percent of Each Grade Level Answering

Each Item on Tests ID and IMC Correctly

Item*		Test ID						Test IMC					
		5	6	7	8	9	10	5	6	7	8	9	10
S1	C1	65	76	71	82	89	88	78	83	92	88	96	96
	C2	75	64	83	85	92	85	88	83	93	93	96	100
	C3	60	68	71	75	86	75	74	89	96	90	94	94
	C4	13	13	18	22	31	28	06	10	13	15	15	31
S2	C1	64	69	69	71	75	85	44	54	72	79	75	83
	C2	64	69	71	83	88	86	42	54	69	79	75	86
	C3	42	58	71	76	83	89	83	88	99	89	94	96
	C4	06	10	07	08	10	14	38	38	25	31	14	22
S3	C1	58	61	65	72	79	82	85	83	82	83	86	78
	C2	58	65	69	85	89	90	82	79	83	82	76	94
	C3	60	63	65	64	88	78	86	81	83	88	92	90
	C4	25	31	31	36	50	42	53	63	58	56	58	67
S4	C1	63	75	72	83	82	75	51	57	65	63	72	79
	C2	03	07	18	24	18	14	11	15	29	22	28	17
	C3	78	89	94	88	94	96	78	86	90	85	90	94
	C4	10	18	29	29	25	25	19	24	54	46	46	44

* In the tables of this chapter the following notation was used to identify solids and cuts: S1 - cube, S2 - triangular prism, S3 - parallelepiped, S4 - cone, S5 - rectangular prism, S6 - cylinder, S7 - four pointed star, S8 - pyramid, C1 - longitudinal cut, C2 - transverse cut, C3 - parallel cut, C4 - oblique cut.

over 70% of a given grade level responded correctly to an item it was considered that subsequent grade levels had also mastered that item.

Of the 16 items on Test I requiring a multiple choice method of response, 8 were answered correctly by over 70% of the fifth grade students. These items involved the longitudinal, transverse, and parallel sections on the cube and the parallelepiped, and the parallel section on the triangular prism and the cone. With the drawing mode of response, only the parallel section on the cone and the transverse section on the cube were represented correctly by over 70% of the fifth grade students in the sample.

By grade 6 over 70% of the students could draw the longitudinal section on the cube and the cone. Just under 70% of this group were successful in drawing the parallel section on the cube, and the longitudinal and transverse sections on the triangular prism. These latter two sections, both of which were rectangles, were correctly selected from the multiple choice distractors by approximately 70% of the seventh grade students.

The parallel section on the triangular prism was first drawn correctly by over 70% of the students at the grade 7 level. At this level 69% of the students could draw the transverse section on the parallelepiped. It was not until grade 8 and 9 respectively that over 70% of the students could represent by drawings the longitudinal and parallel sections on the parallelepiped. These three sections on the parallelepiped were parallelograms and the ability to

draw this figure appears to develop later than the ability to select the figure from a set of distractors.

The longitudinal section on the cone, an isosceles triangle, was not selected from the distractors by more than 70% of the students until grade 9. This was somewhat surprising since the same section was drawn correctly by 75% of the students in the sixth grade.

All items which involved oblique cuts, as well as the transverse cut on the cone, were answered correctly by less than 70% of the students at each grade level tested. Of these items, only the oblique sections on the parallelepiped and cylinder were identified correctly by over 50% of the subjects at any grade level.

Prior to presenting the results for Test II, it should be recalled that different subjects were exposed to a variety of geometric treatments between Test I and Test II. Although none of the treatments included discussion of geometric sections, some or all of the treatments might have affected the student's ability to section solids. For this reason, caution must be exercised in interpreting the results for Test II throughout this report.

The percentage of students at each grade level giving a correct response for each item on Test II is presented in Table 3.

Ten items on Test II were correctly answered by over 70% of the grade 5 subjects. This criterion was met for the longitudinal section on the rectangular prism and the square pyramid, and the parallel section on the cylinder under both

Table 3

Percent of Each Grade Level Answering

Each Item on Tests IID and IIMC Correctly

Item		Test IID						Test IIMC					
		5	6	7	8	9	10	5	6	7	8	9	10
S5	C1	92	90	99	96	100	100	72	93	88	86	92	99
	C2	96	93	97	97	99	99	47	67	61	74	96	94
	C3	90	90	94	94	100	99	63	81	89	85	96	92
	C4	36	43	60	65	75	72	33	53	49	64	81	78
S6	C1	53	67	71	79	93	82	49	65	67	71	92	85
	C2	51	69	69	72	92	82	60	67	78	78	94	89
	C3	94	100	100	96	97	100	90	97	100	96	94	99
	C4	21	39	42	42	68	71	49	68	75	75	86	88
S7	C1	58	79	86	86	90	93	51	65	82	81	86	88
	C2	51	63	58	69	86	68	89	94	96	94	97	96
	C3	58	75	87	83	89	92	44	65	69	85	90	92
	C4	49	67	81	75	79	85	39	60	65	79	86	90
S8	C1	88	97	97	89	93	97	85	94	96	90	93	100
	C2	61	69	75	92	89	85	51	76	76	92	86	90
	C3	54	74	72	79	86	83	71	76	85	82	90	86
	C4	22	31	35	50	57	63	46	51	65	83	89	81

modes of response. The transverse section on the star and the parallel section on the pyramid were selected from sets of distractors. The parallel and transverse sections on the rectangular prism were correctly represented by drawings. These latter two sections were not selected from the sets of distractors by over 70% of the students at a given grade level until grade 6 and grade 8 respectively.

All sections on the rectangular prism were rectangles. As noted in the previous paragraph, these sections were drawn correctly before they were selected from the sets of distractors. This pattern also appeared for several other sections which were rectangles. The longitudinal and parallel sections on the star were first drawn correctly by over 70% of the students in grade 6 whereas the same criterion was not achieved using the multiple choice format until approximately grade 7. The transverse and longitudinal sections on the cylinder indicated a similar 1 year delay from grade 6 to grade 7 and from grade 7 to grade 8 respectively. These same two sections on the triangular prism in Test I also followed this pattern.

With some other sections the opposite trend appeared to occur. Over 70% of the students in grade 5 could select the parallel section on the pyramid, a square, from a set of distractors; however, it was not until a year later that this same criterion was met for the drawing responses. This same phenomenon occurred on Test I where the sections on the cube were squares. The transverse section on the star was not drawn correctly by over 70% of the students until grade 9

compared to its selection in grade 5. This section was star-shaped with an equilateral triangle constructed outward on each side of a square. On Test I where the parallel section on the triangular prism was an equilateral triangle, there was a 2 year delay from grade 5 to grade 7 before 70% of the students could represent the section by a drawing. Figures with several congruent segments appeared to be very difficult for the students to draw.

One other section was represented correctly by grade 6 students. The transverse section on the pyramid was drawn correctly by 69% of the students in grade 6 and selected by 76% from the distractors.

The oblique sections again were the most difficult to recognize; however, the difficulty was not as great for the solids on Test II as for those on Test I. The oblique section on the star, a rectangle, was first drawn correctly by over 70% of the seventh grade students; however, it was not selected from the distractors until 1 year later. For the cylinder where the oblique section was an ellipse, over 70% of the students at the grade 7 level selected the correct drawing but were unable to represent the section by their own drawing until grade 10. The oblique section of the rectangular prism was both drawn and selected from the distractors initially by over 70% of the subjects in grade 9. For the pyramid, 83% of the grade 8 students selected the correct response but at none of the grade levels tested could 70% of the subjects represent the section correctly with a drawing.

Many similarities existed between the results of Test I and those of Test II. These similarities have been noted on the previous pages with careful reference to Tables 2 and 3. They will be discussed further in the next chapter. Further results exploring the relationships between the cuts and the solids follow.

The number of correct responses on each cut and each solid on Test ID and Test IMC are presented in Tables 4 and 5 respectively. Hypothesis 1 was tested for each test using a chi-square test of independence.

> Hypothesis 1: The ability to identify the section resulting from a given cut on a solid is independent of that solid.

For Test ID a chi-square of 294 was obtained with 9 degrees of freedom. This was significant at the .001 level. For Test IMC a chi-square of 274 was found, also with 9 degrees of freedom. This was also significant at the .001 level. Hypothesis 1 was therefore rejected for both Test ID and Test IMC.

Rejection of Hypothesis 1 implied that there was not an inherent difficulty associated with each cut nor with each solid used on Test I. When the totals for each cut are considered independently, the oblique cut was much more difficult than any of the other three cuts. Yet the transverse cut on the cone was more difficult than three of the four oblique cuts on each of Tests ID and IMC. To say that the oblique cut is the most difficult of the four cuts is therefore misleading.

A similar argument exists for the solids. By observing

Table 4

Number of Correct Responses on

Each Cut and Each Solid on Test ID

Cut	Solid S1	S2	S3	S4	Total for Cut
C1	339	312	301	324	1276
C2	348	332	329	60	1069
C3	313	302	300	388	1303
C4	89	39	154	98	380
Total for Solid	1089	985	1084	870	

Maximum N per cell was 432.

Table 5

Number of Correct Responses on

Each Cut and Solid on Test IMC

Cut	Solid S1	S2	S3	S4	Total for Cut
C1	383	295	358	279	1315
C2	397	291	358	88	1134
C3	386	395	373	377	1531
C4	64	119	255	168	606
Total for Solid	1230	1100	1344	912	

Maximum N per cell was 432.

the total number of correct responses for each solid on each test, the cone appeared to be the most difficult of the four solids. Yet the section resulting from the parallel cut on the cone was drawn correctly by more subjects than any other section, and was one of the easiest to select from the distractors. Again, to say that the cone was the most difficult solid is misleading.

Three of the sections on Test I were very difficult for almost all students in the sample. These were the transverse section on the cone, and the oblique section on the cube and the triangular prism.

Only 60 correct drawings were obtained for the transverse section on the cone and only 88 subjects could select the correct figure, one nappe of a hyperbola, from the distractors. The most common error with this section was to ignore the curvature of the sides of the hyperbola. Many subjects also represented the top of the figure as pointed rather than curved.

The oblique cut on the cube resulted in a rectangle as the section. The most common response for this section was the drawing or selection of a square. Subjects appeared unable to overcome their perception of the cube, every side of which was a square.

The most difficult section to draw was the oblique section on the triangular prism. This section was an isosceles triangle with the vertical angle approximately equal to 90°. Due to the orientation of the prism and the direction of the oblique cut used in this study, it was not

possible for any oblique section on this solid to be an isosceles triangle with a vertical angle less than 60°. If the orientation of the solid were changed slightly, say a 30° turn about the vertical axis, the oblique cut would have resulted in a section which was an isosceles triangle whose vertical angle could not have been greater than 60°. Many subjects cither drew or chose an isosceles triangle with a small vertical angle and this was marked incorrect. In the multiple choice format, none of the figures were correct resulting in "none of thcse" being the correct answer. This may have caused additional difficulty.

Hypothesis 1 was also tested for Test IID and Test IIMC. The number of correct responses for each cut and each solid on these two tests are presented in Tables 6 and 7 respectively.

For Test IID a chi-square of 77.9 with 9 degrees of freedom was found and for Test IIMC a chi-square of 46.9, again with 9 degrees of freedom, was reportcd. Both were significant at the .001 level. Hypothesis 1 was therefore rejected for each of the two tests.

As was the case with Test I, rejection of Hypothesis 1 for Tests IID and IIMC implied that there was not an inherent difficulty associated with each cut nor with each solid used on Test II. The oblique cut was again by far the most difficult when the total number of responses for each cut were considered. Yet the transverse section on the star was more difficult to represent by a drawing than the oblique cut on that solid. Many students had difficulty representing

Table 6

Number of Correct Responses on

Each Cut and Solid on Test IID

Cut	Solid S5	S6	S7	S8	Total for Cut
C1	415	320	355	404	1494
C2	418	314	285	339	1356
C3	409	424	349	323	1505
C4	253	203	313	185	954
Total for Solid	1495	1261	1302	1251	

Maximum N per cell was 432.

Table 7

Number of Correct Responses on

Each Cut and Solid on Test IIMC

Cut	Solid S5	S6	S7	S8	Total for Cut
C1	381	308	326	402	1417
C2	317	335	407	340	1399
C3	362	415	321	353	1451
C4	257	318	302	299	1176
Total for Solid	1317	1376	1356	1394	

Maximum N per cell was 432.

the star-shaped transverse section whereas the oblique section, which was a rectangle, was perhaps easier to draw. On Test IIMC this transverse section was very easy to select from the distractors with 407 correct responses whereas the oblique section was only recognized correctly 302 times.

On Test IIMC the oblique section on the cylinder, an ellipse, was marginally easier to recognize than was the longitudinal section, a rectangle. However, on the Test IID the ellipse was drawn correctly only 203 times compared to 320 correct drawings of the rectangle.

These two examples, while illustrating the interaction of the cut and solid, also suggest that the mode of response was a significant factor. Some sections, such as the star or ellipse, might have been correctly represented by the child in his or her mind, but were not represented correctly as a drawing.

Previous studies by Boe (1966), Davis (1969), and Pothier (1975) used the cube, cylinder, rectangular prism, and cone on their tests. The average total score on these four solids for each of grades 6, 8, and 10 was found for the drawing (D) and multiple choice (MC) modes of response. In the present study the cube and cone were used on Test I and the rectangular prism and cylinder on Test II. The data were therefore collected on two separate occasions. Also it is recalled from Chapter III that the various studies differed in several aspects with regard to the orientation of the solids and the administration of the test. With these limitations in mind, the means for the three previous studies

together with those for the present study are given in Table 8.

Table 8

Comparisons of Means of Sections on the Cube, Cone, Rectangular Prism, and Cylinder with Those Obtained in Previous Studies

	Grade					
	6		8		10	
Study	D	MC	D	MC	D	MC
Boe	-	-	11.4	10.4	11.8	11.2
Davis	-	10.5	-	12.7	-	13.5
Pothier	10.7	10.7	11.9	11.5	13.0	13.1
This Study	10.0	10.4	11.3	11.3	11.9	13.8

When compared with Boe's (1966) results, the means obtained at the grade 8 and grade 10 levels on the drawings are almost identical to those obtained in this study. In the multiple choice format, the present results were somewhat higher, particularly at the grade 10 level where the difference in means was 2.6. Davis (1969) used only the multiple choice mode of response and the means for grades 6 and 10 are very similar to those from the present study whereas the results at grade 8 showed Davis' subjects performing slightly better.

Pothier's (1975) study, also conducted in Edmonton, yielded results at each of grades 6, 8, and 10 which were marginally higher than those obtained in this study. The

one exception to this was at grade 10 where her students did not perform as well on the multiple choice portion of the test. Overall the differences in results on the four studies are very small. This gives some indication that variations in the testing procedure did not appear to affect the results very much.

Question 2

Are there differences in the ability of students of a particular sex, grade level, and ability to section solids?

A 6 x 2 x 3 factorial design was used to answer this question with the dependent variables being grade level, sex, and ability respectively. The means for each of the 36 cells on each of Tests ID, IMC, IID, and IIMC are presented in Tables 9, 10, 11, and 12 respectively. The mean profiles for each of the three variables on the four tests are presented in Figure 3. Observation of the three graphs suggested that significant effects were present for grade level, sex, and ability. Hypotheses 2a, 2b, and 2c were tested using MANOVA to investigate these possibilities.

> Hypothesis 2a: There is no significant difference between the grade mean vectors on Tests ID, IMC, IID, and IIMC.
>
> Hypothesis 2b: There is no significant difference between the sex mean vectors on Tests ID, IMC, IID, and IIMC.
>
> Hypothesis 2c: There is no significant difference between the ability mean vectors on Tests ID, IMC, IID, and IIMC.

The results of the MANOVA are presented in Table 13. All three main effects yielded F-ratios which were significant at the .001 level. However, the grade by

Table 9

Cell Means on Test ID

Ability	Sex	Grade 5	6	7	8	9	10	$\bar{X}_{Ability}$
L	M	6.92	7.08	7.50	7.58	10.2	10.3	7.60
	F	5.92	6.67	5.42	7.00	8.83	7.75	
A	M	6.83	8.08	9.25	10.0	11.1	10.0	9.29
	F	5.92	8.58	9.50	10.6	10.9	10.8	
H	M	9.42	9.75	11.6	11.9	12.1	11.9	11.1
	F	9.50	9.92	11.0	12.0	11.5	12.3	
$\bar{X}_{Grade}$		7.42	8.35	9.04	9.85	10.8	11.5	
$\bar{X}_{Sex}$			$\bar{X}_{Male} = 9.53$			$\bar{X}_{Female} = 9.11$		

Table 10

Cell Means on Test IMC

Ability	Sex	Grade 5	6	7	8	9	10	$\bar{X}_{Ability}$
L	M	8.67	8.75	9.83	10.3	11.0	10.8	9.36
	F	7.58	8.00	9.83	8.50	8.33	10.7	
A	M	8.75	11.2	11.6	10.8	11.8	11.7	10.9
	F	8.25	10.1	11.4	11.3	11.5	12.3	
H	M	11.1	10.8	12.3	12.4	12.3	12.6	11.6
	F	10.6	10.5	11.3	12.1	11.6	11.8	
$\bar{X}_{Grade}$		9.15	9.89	11.0	10.9	11.1	11.6	
$\bar{X}_{Sex}$			$\bar{X}_{Male} = 10.9$			$\bar{X}_{Female} = 10.3$		

Table 11

Cell Means on Test IID

Ability	Sex	Grade 5	6	7	8	9	10	$\bar{X}_{Ability}$
L	M	9.17	10.3	11.3	10.9	14.3	13.4	10.7
	F	8.33	8.75	9.42	9.67	11.7	10.9	
A	M	8.17	12.0	12.7	12.8	14.2	14.1	12.3
	F	8.33	12.1	12.7	13.0	13.7	13.3	
H	M	12.9	12.4	13.8	14.8	15.3	15.3	13.9
	F	11.4	13.1	13.5	14.8	14.6	15.2	
$\bar{X}_{Grade}$		9.72	11.4	12.2	12.7	13.9	13.7	
$\bar{X}_{Sex}$			$\bar{X}_{Male}$ = 12.7			$\bar{X}_{Female}$ = 11.9		

Table 12

Cell Means on Test IIMC

Ability	Sex	Grade 5	6	7	8	9	10	$\bar{X}_{Ability}$
L	M	8.08	10.2	10.9	12.9	14.0	14.2	10.8
	F	7.83	8.08	9.08	8.50	12.7	12.6	
A	M	8.83	12.9	13.3	13.9	15.1	14.7	12.8
	F	6.92	12.3	13.3	13.2	14.6	14.8	
H	M	14.2	14.1	15.3	10.5	13.8	15.6	14.3
	F	13.9	15.3	15.0	13.3	14.8	15.4	
$\bar{X}_{Grade}$		9.39	11.8	12.4	13.1	14.5	14.4	
$\bar{X}_{Sex}$			$\bar{X}_{Male}$ = 13.2			$\bar{X}_{Female}$ = 12.1		

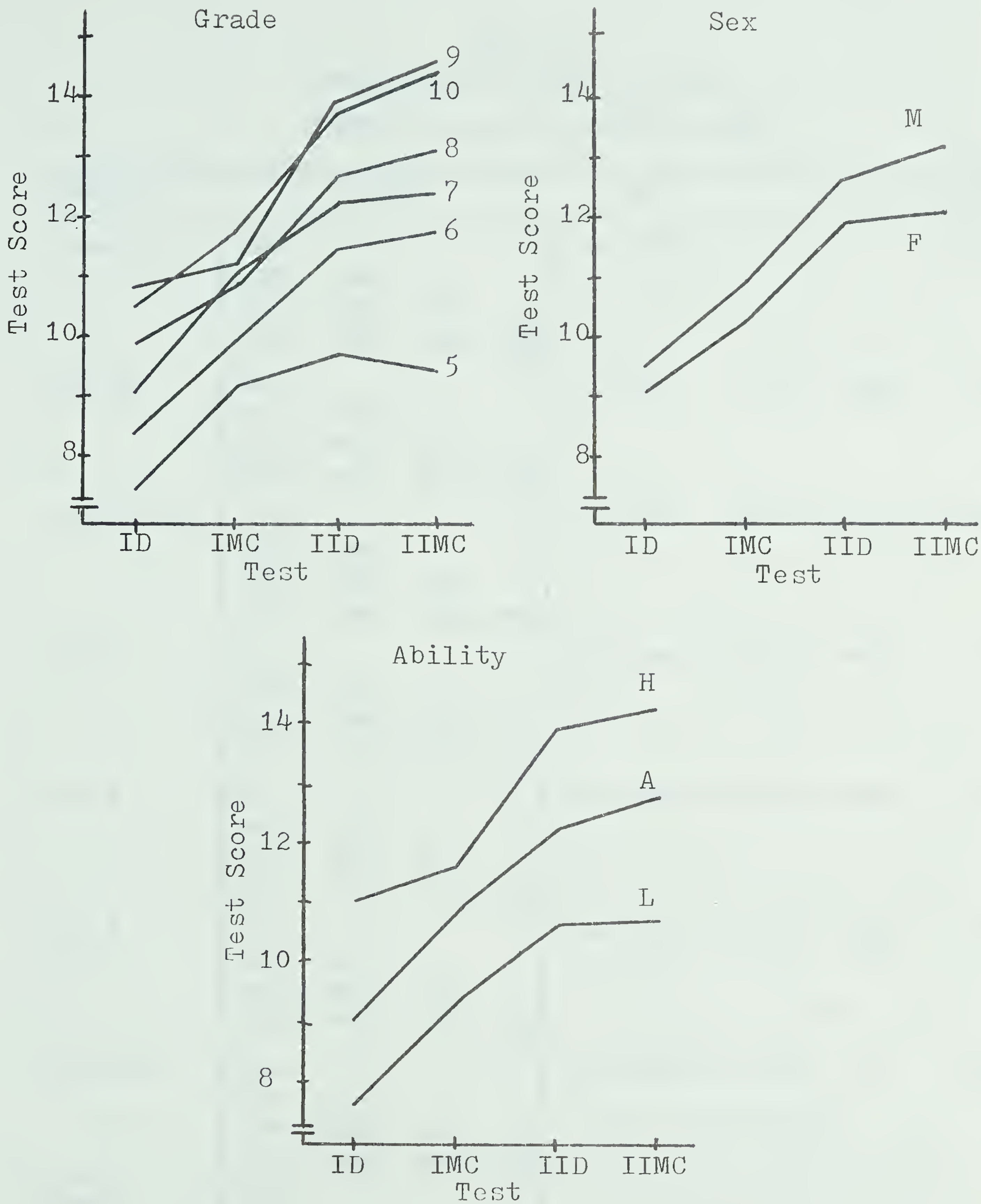


Figure 3

Mean Profiles by Grade, Sex, and Ability for Tests ID, IMC, IID, and IIMC

Table 13

Multivariate Analysis of Variance

(MANOVA) for Tests ID, IMC, IIC, IIMC

Source	SSP				df	F	Prob	Wilk's λ
Grade (G)	605				20, 1304.4	10.3	0	.614
	389	302						
	717	478	875					
	880	588	1080	1330				
Sex (S)	18.8				4, 393	5.18	.0005	.950
	28.1	42.2						
	33.3	50.0	59.3					
	48.8	73.1	86.7	127				
Ability (A)	868				8, 786	23.7	0	.649
	561	380						
	813	525	761					
	879	582	823	901				
G x S	13.0				20, 1304.2	1.51	.069	.927
	.458	12.2						
	10.9	1.62	15.1					
	-10.8	3.10	-16.8	43.9				
S x A	46.5				8, 786	1.58	.126	.969
	24.9	15.6						
	51.7	27.4	57.5					
	41.9	81.8	46.6	37.9				
G x A	116				40, 1492.1	1.67	.0058	.847
	41.1	59.3						
	99.6	72.2	139					
	114	97.9	157	226				
G x S x A	25.1				40, 1492.1	1.16	.234	.891
	-6.79	33.6						
	1.60	-3.22	20.6					
	5.79	5.73	28.9	97.3				
Error	2150							
	1090	1640						
	1230	957	2400					
	1130	941	1800	2830				

ability interaction was also significant at the .01 level. Although the grade by sex and sex by ability interactions were not significant at the .05 level, their possibilities of occurring were great enough that further examination of these interactive effects was deemed necessary prior to any interpretation of the main effects.

Since the results from the MANOVA indicated significant interactions and main effects on the vector means of Tests ID, IMC, IID, and IIMC, a three-way ANOVA was conducted on each of the four tests separately. The results of these ANOVA's are reported in Tables 14, 15, 16, and 17.

The results from the MANOVA indicated that an hypothesis of no significant grade by sex interaction would be rejected at only the .069 level of significance. The probability levels obtained from the separate ANOVA's indicated that this hypothesis was tenable and should not be rejected for any of the tests. Graphs of the grade by sex interaction on each test are shown in Figure 4. These graphs illustrate that on Tests IMC, IID, and IIMC, males scored higher than females at every grade level. On Test ID, males scored higher than females at each grade level except grades 6 and 8 where the mean scores were almost the same.

On each of Tests ID, IID, and IIMC, performance increased with each grade level for both males and females with the exception that grade 9 students scored as high or higher than grade 10 students. On Test IMC the pattern was similar except that grade 8 students of both sexes did not score as high as grade 7 students and grade 9 females scored

Table 14

Analysis of Variance for Test ID

Source	SS	df	MS	F	Prob
Grade (G)	606	5	121	22.3	0
Sex (S)	18.8	1	18.8	3.45	.06
Ability (A)	868	2	434	80.0	0
G x S	13.0	5	2.59	.48	.79
S x A	46.5	2	23.3	4.28	.01
G x A	116	10	11.6	2.14	.02
G x S x A	25.1	10	2.51	.46	.91
Error	2150	396	5.43		

Table 15

Analysis of Variance for Test IMC

Source	SS	df	MS	F	Prob
Grade (G)	302	5	60.5	14.6	0
Sex (S)	42.2	1	42.2	10.2	.002
Ability (A)	380	2	190	46.0	0
G x S	12.2	5	2.45	.59	.71
S x A	15.6	2	7.80	1.89	.15
G x A	59.3	10	5.93	1.44	.16
G x S x A	33.6	10	3.36	.81	.62
Error	1640	396	4.13		

Table 16

Analysis of Variance for Test IID

Source	SS	df	MS	F	Prob
Grade (G)	875	5	175	28.9	0
Sex (S)	59.3	1	59.3	9.78	.002
Ability (A)	761	2	380	62.8	0
G x S	15.1	5	3.02	.50	.78
S x A	57.5	2	28.8	4.75	.009
G x A	139	10	13.9	2.30	.012
G x S x A	20.6	10	2.06	.34	.97
Error	2400	396	6.06		

Table 17

Analysis of Variance for Test IIMC

Source	SS	df	MS	F	Prob
Grade (G)	1330	5	266	37.3	0
Sex (S)	127	1	127	17.8	0
Ability (A)	901	2	451	63.2	0
G x S	43.9	5	8.78	1.23	.29
S x A	37.9	2	18.9	2.65	.072
G x A	226	10	22.7	3.17	.0006
G x S x A	97.3	10	9.73	1.36	.20
Error	2830	396	7.14		

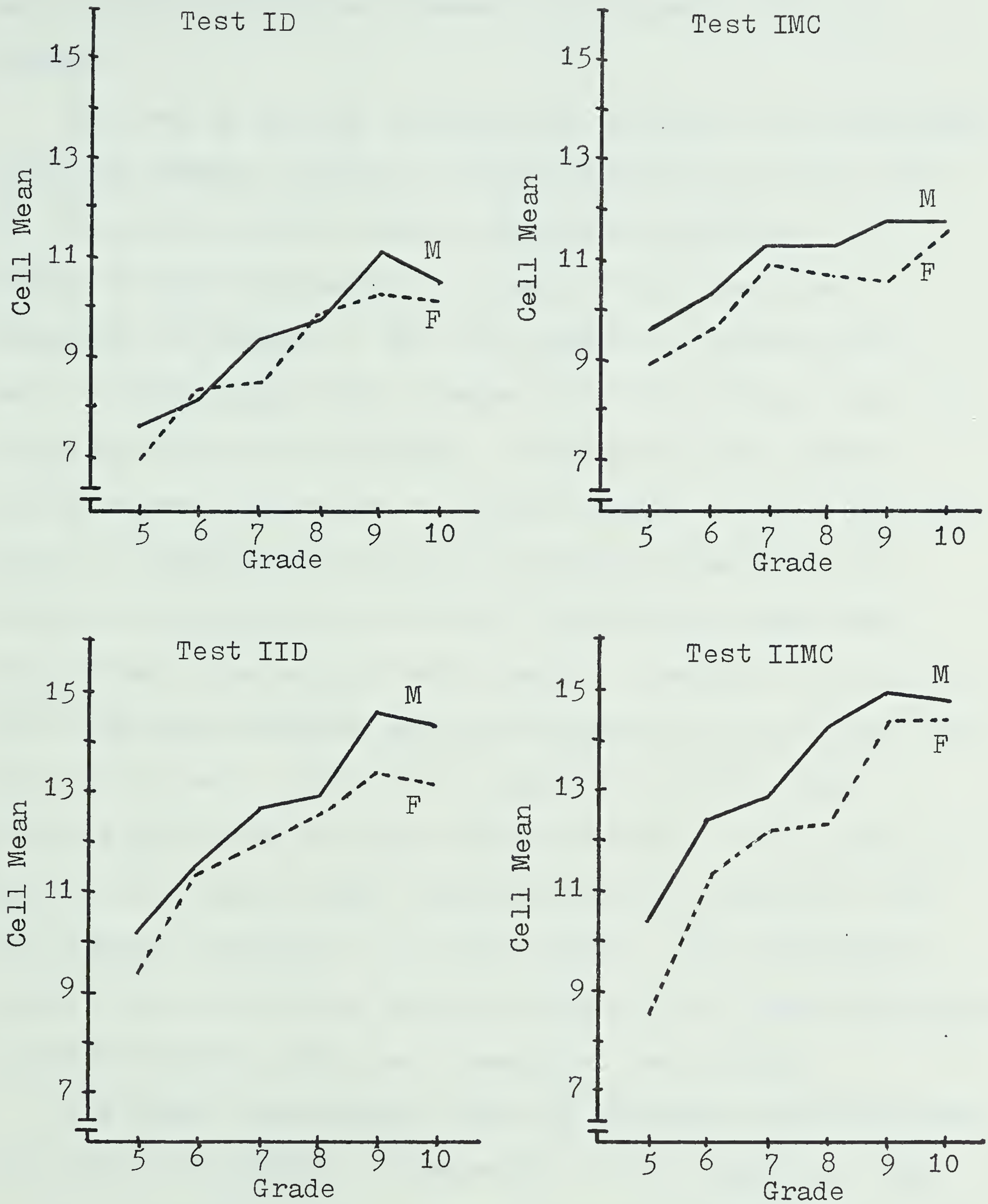


Figure 4

Grade by Sex Interactions on Tests ID, IMC, IID, IIMC

still lower than grade 8 females. The lines in each graph presented in Figure 4 are thus close to being parallel and very little interaction between sex and grade level is evident.

The sex by ability interaction, although not significant using the MANOVA analysis, yielded significant results for both Tests ID and IID with the individual ANOVA's. The graphs of this interaction for each of four tests are presented in Figure 5. The four graphs are similar with males scoring higher than females at every ability level on every test with one exception. Average ability females scored higher than males of similar ability on Test ID. The source of interaction was due to the fact that high and average ability males scored only marginally higher than their female counterparts with the one exception noted above. The differences between males and females of low ability were more pronounced. High ability students of both sexes scored higher than average ability students of the same sex who in turn scored higher than low ability students. This same pattern occurred on all four tests. The interaction between sex and ability therefore was due to large differences in scores between males and females of low ability.

The third interaction, grade by ability, was significant ($p < .01$) on the MANOVA. Observation of the results of the individual ANOVA's revealed this interaction significant for Test ID ($p < .05$), Test IID ($p < .05$), and Test IIMC ($p < .001$). Graphs of the grade by ability relationships on all four tests are presented in Figure 6.

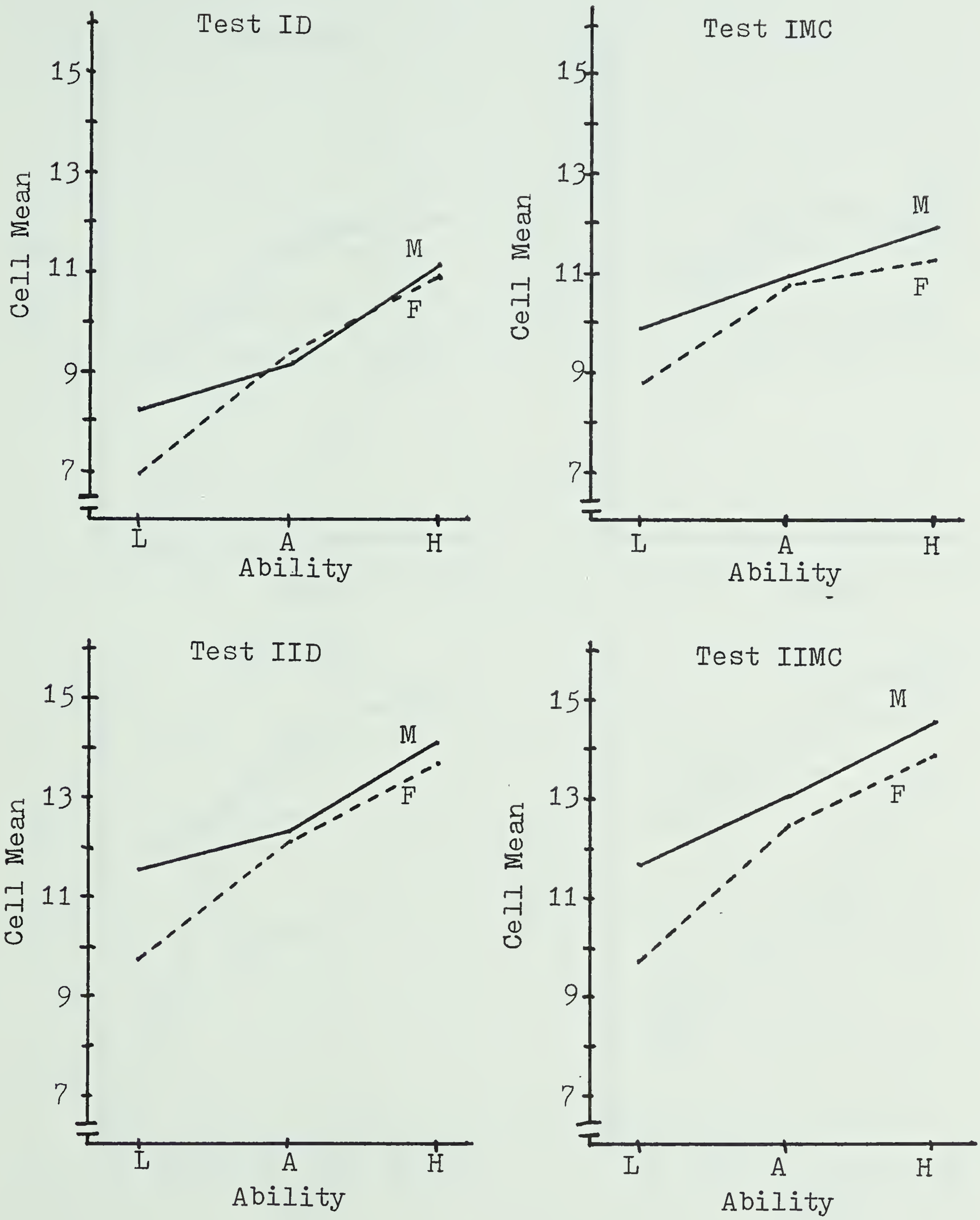


Figure 5

Sex by Ability Interactions on Tests ID, IMC, IID, IIMC

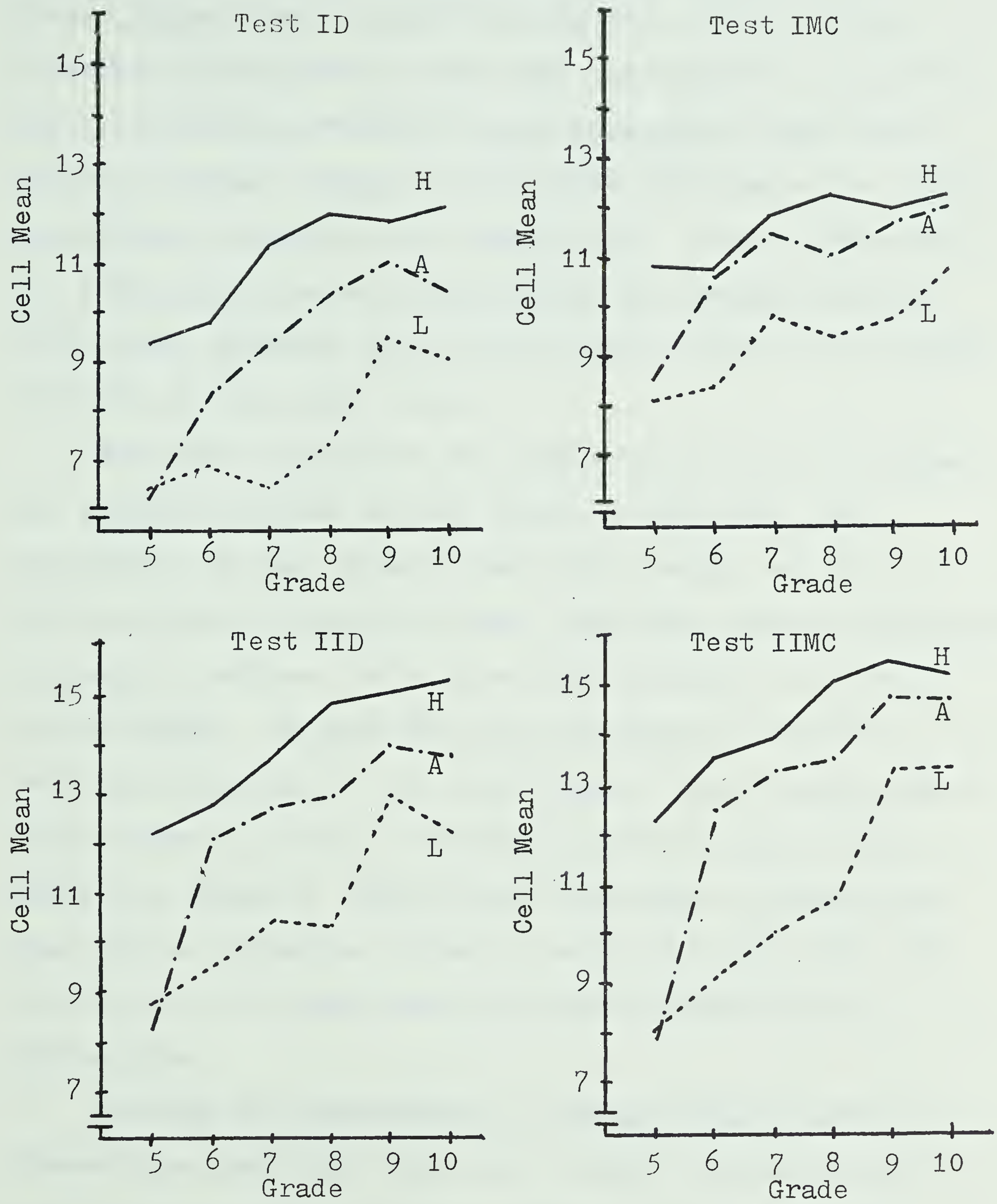


Figure 6

Grade by Ability Interactions on Tests ID, IMC, IID, IIMC

On each of the four tests, high ability students scored better than those of average ability who in turn scored higher than low ability students. This pattern occurred at every grade level with the exception of grade 5 where low ability students scored marginally higher than those of average ability on the three tests where the grade by ability interaction was significant. On Test IMC where the interaction was not significant, the average ability fifth grade students also scored higher than the low ability students at that grade level.

With some exceptions the scores on all tests increased for students at each ability level as the grade level increased. On each drawing test both average and low ability grade 10 students scored lower than their counterparts in grade 9; whereas the high ability students in grade 10 scored higher. On Test IMC the same pattern occurred between grades 7 and 8. On both forms of Test I there was a slight drop in scores of the high ability students from grade 8 to grade 9. These drops from grade to grade were small in all cases and tended to occur with more than one ability level at each grade level thus minimizing any interaction.

Although the performance of average ability grade 5 students was very low, there was a sharp increase on all tests for this group between grades 5 and 6. A similar increase was evident for students of low ability between grades 8 and 9, especially on Tests ID, IID, and IIMC. Other portions of the graphs had very similar slopes for each

ability level. The grade by ability interaction therefore appeared to be a result of these sharp increases in performance.

The main effect of grade level was found to be a significant factor on each of the four tests as well as the vector of means on these tests. The grade by sex interaction was not significant for any of the tests. For each sex, the means increased as the grade level increased except between grades 9 and 10. The grade by ability interaction, although significant, was due mainly to the increase in performance of average ability students between grades 5 and 6, and of low ability students between grades 8 and 9. Again, the means increased for each ability level as the grade level increased, with some exceptions, especially between grade 9 and grade 10.

Hypothesis 2a was rejected and the alternate hypothesis that there are significant differences between the grade mean vectors was accepted. The individual ANOVA's further revealed that grade level was a significant factor on all four tests.

Using the procedure suggested by Bay (1969b) within the multivariate analysis, comparisons were made between the performances at each grade level. The F-ratios are presented in Table 18.

Grades 8, 9, and 10 all scored significantly higher than grade 5 on each of the four tests with the exception of the comparison between grade 8 and grade 5 on Test IMC. Grade 7 students also scored significantly higher than the

Table 18

F-Ratios for Multiple Comparisons of Grade Levels for

Tests ID, IMC, IID, IIMC Following MANOVA and ANOVA Analyses

	ID		IMC		IID		IIMC	
Comparison	MANOVA	ANOVA	MANOVA	ANOVA	MANOVA	ANOVA	MANOVA	ANOVA
10 - 9	.02	.09	.13	.54	.02	.07	.00	.01
10 - 8	.14	.57	.23	.95	.32	1.29	.44	1.79
10 - 7	.69	2.82*	.15	.62	.62	2.53*	1.01	4.15**
10 - 6	1.49	6.15**	1.30	5.34**	1.46	6.02**	1.75*	7.25**
10 - 5	2.97**	12.6**	2.56**	10.8**	4.31**	18.8**	5.79**	25.8**
9 - 8	.28	1.12	.01	.06	.49	1.98	.51	2.07
9 - 7	.96	3.94**	.00	.00	.85	3.47**	1.11	4.56**
9 - 6	1.86*	7.75**	.61	2.49*	1.79*	7.43**	1.89*	7.79**
9 - 5	3.47**	14.9**	1.57*	6.50**	4.83**	13.3**	5.99**	26.8**
8 - 7	.21	.86	.01	.03	.05	.21	.12	.49
8 - 6	.73	2.98*	.44	1.79	.43	1.74	.45	1.83
8 - 5	1.88*	7.84**	1.30	5.34**	2.43*	12.2**	3.28*	14.0**
7 - 6	.16	.64	.57	2.32*	.18	.75	.11	.43
7 - 5	.83	3.50**	1.50	6.22**	1.81*	7.51**	2.21**	9.25**
6 - 5	.28	1.15	.23	.95	.86	3.53**	1.38	5.69**

* $p < .05$

** $p < .01$

grade 5 students on Tests IID and IIMC. The level of significance was .01 for all comparisons involving grades 9 and 10 with grade 5 except for the grades 5 and 9 comparison on Test IMC. All other comparisons above were significant at the .05 level. This level of significance was also attained for the comparisons between grade 9 and grade 6 on Tests ID, IID, and IIMC.

The procedure for carrying out multiple comparisons on multivariate data is much more conservative than using a similar procedure on each of the variables separately. The F-ratios for the multiple comparisons done separately on each test are also presented in Table 18.

Under this more liberal procedure every other grade level scored significantly higher than grade 5 on each of the four tests except grade 6 on Tests ID and IMC. The level of significance was less than .01 in every case. Grades 9 and 10 also scored significantly higher than grade 6 on all tests, and higher than grade 7 students on all tests except Test IMC. The level of significance was less than .01 in all cases except the comparison betwen grades 10 and 7 on Tests ID and IID and the comparison between grades 9 and 6 on Test IMC. Here the significance level was less than .05.

Two other comparisons were significant at the .05 level. Grade 8 students scored higher than grade 6 students on Test ID and grade 7 students higher than grade 6 students on Test IMC.

To summarize the above, two conclusions may be stated with reasonable certainty. First, students in both grades

9 and 10 scored higher on the sectioning tests than did students in either grade 5 or 6. Secondly, students in grade 5 in the sample employed in this study were less successful on the sectioning tasks than students at succeeding grade levels. The very low performance of average grade 5 students may account for the latter result.

Neither interaction involving sex was significant using the MANOVA procedure. When ANOVA was carried out on each test, the sex by ability interaction was significant on Tests ID and IID. Except for Test ID, where average ability females scored marginally higher than males, the interactions were ordinal. Hypothesis 2b was therefore rejected. Males scored higher than females on all four tests, the differences being significant ($p < .01$) in each case except on Test ID. Here the difference was not significant, the probability level being only .06. It should be noted that for students of low ability the difference was somewhat greater than at the higher ability levels. The sex difference, although statistically significant, accounted for only a small portion of the variance of each test. The educational significance of this difference therefore is minimal, especially for students of average and high ability.

As noted previously the third main effect, ability level, interacted significantly with both sex and grade level on several of the tests. The interaction with sex was ordinal, except as noted above. Interaction with grade was due mainly to sharp increases for students of average and low ability from grade 5 to grade 6 and from grade 8 to grade 9

Table 19

F-Ratios for Multiple Comparisons of Ability Levels for Tests ID, IMC, IID, and IIMC Following MANOVA and ANOVA Analyses

Comparison	ID		IMC		IID		IIMC	
	MANOVA	ANOVA	MANOVA	ANOVA	MANOVA	ANOVA	MANOVA	ANOVA
H - A	5.07**	21.0**	1.14	4.64*	4.05**	16.7**	2.63**	10.7**
H - L	18.2**	80.0**	10.4**	44.1**	14.5**	62.8**	14.5**	62.5**
A - L	4.62**	19.0**	4.88**	20.2**	3.60**	14.8**	5.19**	21.5**

* $p < .05$

** $p < .01$

respectively. Although this latter interaction posed some limitation on the interpretation of the ability main effect, multiple comparisons were conducted on each pair of ability levels. The F-ratios are reported in Table 19. Using the MANOVA comparisons, high ability students scored higher than those of average ability who in turn scored higher than those of low ability. All comparisons were significant ($p < .01$) except the high-average comparison on Test IMC. Using ANOVA comparisons, the results were the same, except the high-average comparison on Test IMC was significant ($p < .05$). Except for grade 5 where there was little difference between students of low and average ability it was concluded that ability to section solids increased with the ability level of the student. Hypothesis 2c was therefore rejected.

Question 3

Is the ability to section solids a uni-factor trait or is it a composite of several independent or related abilities?

Using the entire sample of 432 subjects, correlations were found between each item on Test I and are reported in Tables 39, 40, and 41 in Appendix 3. Table 39 contains the correlations among items on Test ID; Table 40, the correlations among items on Test IMC; and Table 41, the correlations between items on Tests ID and IMC. The correlations range from -.14 to .53. Any correlation whose absolute value is greater than or equal to .13 is significantly different from zero ($p < .01$) for a sample of

432. Although many of the correlations met this criterion, few exceeded .30. The variance common to items was therefore very low in almost every instance.

Despite the low correlations, it was felt that it was desirable to proceed with the principal axis factoring since this portion of the study was exploratory in nature. The correlation matrix was therefore subjected to principal axis factoring, the factors extracted being rotated by the varimax procedure to approximate orthogonal simple structure. Squared mean correlations were used in the diagonal as estimates of the communalities.

Although only one eigen value greater than one was found, the choice of four factors seemed to provide the most meaningful interpretation. This four factor solution is presented in Table 20. The communalities among these four factors accounted for only 22.3% of the total variance; however, this was expected due to the lack of high entries in the correlation matrix.

Using the criterion of .30 as the minimum absolute value for a factor loading to be interpreted, the first factor was found to have 13 such loadings. The longitudinal and parallel sections on the cube and on the cone, and the transverse and parallel sections on the triangular prism were included in these 13 under both the drawing and multiple choice responses. No apparent connection existed among these items and therefore this factor could represent a general sectioning ability. This is a very liberal interpretation of this factor since it accounted for only

Table 20

Principal Axis Factoring Under

Varimax Rotation for Test 1 - Entire Sample

	Variable		Factor 1	2	3	4	Communalities
Drawing Responses	S1	C1	35	24	19	18	25
		C2	24	12	24	14	15
		C3	35	17	27	11	24
		C4	02	27	09	17	11
	S2	C1	12	60	07	02	38
		C2	31	40	04	15	28
		C3	30	03	16	27	19
		C4	06	05	-08	-01	01
	S3	C1	22	17	41	19	28
		C2	16	19	51	15	34
		C3	06	19	50	12	30
		C4	13	19	36	04	18
	S4	C1	42	13	09	03	20
		C2	08	19	07	34	16
		C3	49	20	13	13	32
		C4	13	08	09	45	23
Multiple Choice Responses	S1	C1	52	06	12	15	31
		C2	30	14	27	-02	19
		C3	54	02	25	11	36
		C4	01	26	06	08	08
	S2	C1	27	65	09	12	52
		C2	37	52	10	19	45
		C3	46	03	04	12	23
		C4	-13	-04	06	-17	05
	S3	C1	12	-10	27	09	10
		C2	09	06	17	-07	05
		C3	15	01	36	-08	16
		C4	19	04	10	-09	06
	S4	C1	42	31	15	02	29
		C2	-09	08	02	35	14
		C3	46	16	18	06	28
		C4	18	09	05	45	25
Eigen values			4.87	.91	.69	.66	
Variance			2.64	1.88	1.52	1.09	
% Total Variance			8.25	5.86	4.75	3.40	
% Common Variance			37.1	26.3	21.3	15.3	

Sum of Communalities = 7.12

Total Variance Accounted for = 22.3%

8.25% of the total variance.

The second factor had high loadings on the longitudinal and transverse sections of the triangular prism under both modes of response. These two sections were very similar in that they were both rectangles. The only other rectangular section resulted from the oblique cut on the cube. The loadings for that section were just less than the .30 criterion. The only other significant loading on this factor was the multiple choice response to the longitudinal cut on the cone.

The third factor had loadings greater than .30 on all four sections on the parallelepiped using the drawing method of response. The only other "high" loading was for the parallel sections on the parallelepiped with the multiple choice response. Since only the sections involving the parallelepiped were nonrectangular parallelograms, this factor appeared to reflect the ability to recognize these figures, or more specifically to draw nonrectangular parallelograms.

The four significant loadings on the fourth factor were the two forms of each of the transverse and oblique sections on the cone. These two sections were one nappe of a hyperbola and an ellipse respectively. The ability to represent these conics thus was the main characteristic of this factor.

As mentioned previously caution must be exercised in the interpretation of these factors due to the low percentage of variance for which each accounted.

A correlation matrix was also determined for the items on Test II using the sample of 432 subjects. Table 42 contains the correlations among the items on Test IID; Table 43 the correlations among the items on Test IIMC; and Table 44 the correlations between the items on Test IID and those on Test IIMC. All three tables are found in Appendix 3.

The correlations for Test II, although still low, were marginally higher than for Test I, they varied from -.04 to .73. Again any correlation greater than or equal to .13 was significantly different from zero ($p < .01$). As was the case for Test I only a few correlations exceeded .30.

The correlation matrix was again factored using principal axis factoring with varimax rotation. Two eigen values were found greater than unity, however the five factor solution appeared to be the most interpretable. That solution accounted for 35% of the common variance and is presented in Table 21.

The first factor contained loadings greater than .30 on both modes of response for the oblique section of the rectangular prism, the longitudinal and transverse sections of the cylinder, and the parallel section on the pyramid. In addition the longitudinal section on the star with the drawing response, and the transverse section on the rectangular prism and oblique section on the star with the multiple choice response also loaded on this factor. All of these sections were rectangles except the parallel section on the pyramid which was a square, the only square on Test II.

Table 21
Principal Axis Factoring Under
Varimax Rotation for Test II - Entire Sample

	Variable		Factor 1	2	3	4	5	Communalities
Drawing Responses	S5	C1	12	12	14	14	19	10
		C2	06	14	11	-03	15	06
		C3	09	18	02	16	09	07
		C4	36	34	24	11	27	39
	S6	C1	69	20	12	02	08	54
		C2	74	15	11	04	13	60
		C3	-03	05	-03	18	09	05
		C4	12	16	10	11	54	35
	S7	C1	37	26	46	12	05	44
		C2	20	31	12	10	15	18
		C3	28	19	49	24	27	48
		C4	27	36	24	26	11	34
	S8	C1	-02	07	10	26	09	09
		C2	15	55	13	13	12	37
		C3	32	20	22	22	19	28
		C4	21	50	17	11	12	35
Multiple Choice Responses	S5	C1	14	08	14	30	19	17
		C2	30	13	25	25	23	29
		C3	03	08	31	34	20	26
		C4	41	34	20	14	29	44
	S6	C1	69	24	31	13	16	67
		C2	68	30	27	01	08	63
		C3	23	05	08	49	-04	30
		C4	10	10	10	09	54	33
	S7	C1	29	29	63	15	08	60
		C2	-05	20	04	42	00	22
		C3	28	26	62	16	24	61
		C4	33	39	49	14	19	55
	S8	C1	18	10	14	62	-03	45
		C2	14	46	17	23	09	32
		C3	37	09	28	21	32	36
		C4	16	50	13	19	10	33
Eigen values			8.22	1.15	.69	.62	.54	
Variance			3.46	2.32	2.32	1.72	1.40	
% Total Variance			10.8	7.26	7.24	5.37	4.37	
% Common Variance			30.8	20.7	20.7	15.3	12.5	

Sum of Communalities = 11.2
Total Variance Accounted for = 35.0%

Although several other sections on Test II were rectangles, it appeared this first factor was related to the ability to represent the rectangular sections.

The oblique sections from both drawing and multiple choice modes of response for the rectangular prism, star, and pyramid all loaded on the second factor. These sections on the prism and star were rectangles whereas the oblique section on the pyramid was an isosceles trapezoid. The only other section which was trapezoidal in shape, the transverse section on the pyramid, also loaded on this factor. The other two loadings greater than .30 were the drawing of the transverse section on the star, a star shape, with a loading of .31 and the multiple choice selection of the same cut on the cylinder, a rectangle, with a loading of .30. This factor could be associated with the oblique cuts, however the four highest loadings were for the items involving the trapezoid.

The fourth oblique cut, that on the cylinder, resulted in an ellipse as a section. Both items involving this section loaded on the fifth factor, the two loadings being .54. There was only one other loading on this factor meeting the criterion, that for the multiple choice response to the parallel item on the pyramid with a loading of .32. This factor appears to be similar to the fourth factor on Test I where the oblique cut on the cone, also an ellipse, and the transverse cut on the same solid contributed the significant loadings. It seems more reasonable then to accept an interpretation of factor two above as one due to the trapezoid rather than the oblique cut.

On the third factor loadings greater than .45 were found for the longitudinal and parallel section on the star under both modes of response and for the oblique section on the same solid with the multiple choice response. The loading for the drawing response of the oblique section was .24. These sections were all rectangles. The transverse section on this solid was star shaped and had loadings of .12 and .04 on this factor. The factor therefore appeared to be determined by the rectangular section resulting from the longitudinal, parallel, and oblique cuts on the star. Two other sections, again both rectangles, had loadings on this factor of .31. This was quite low in comparison to the loadings for the star.

Factor four on Test II had five loadings greater than .30. All were from the multiple choice method of response and no apparent relationship existed among the items. This was the only case which suggested the possibility of some difference in abilities being required for the two modes of response.

The preceding analysis was conducted to arrive at a decision regarding Hypothesis 3.

> Hypothesis 3: The ability to section solids is a uni-factor trait.

Since the correlations between sectioning items were low, less than .30 in most instances, and since the factor analysis, although accounting for only a small portion of the total variance, suggested the possible existence of several factors, Hypothesis 3 was therefore rejected.

Briefly, it appears that method of response is not a factor since in most instances both items involving a particular section loaded on the same factor. The type of section appeared to determine upon which factor an item loaded in several instances, for example rectangles, parallelograms, trapezoids, and ellipses. The solid, particularly the parallelepiped and the star also appeared to influence the loadings. Since any interpretation of these results must be somewhat subjective, further discussion is reserved for the concluding chapter.

Principal axis factoring was also carried out on Test I and Test II separately for each sex, for each ability level and for grades 5 and 6 combined, grades 7 and 8 combined, and grades 9 and 10 combined. In every case the correlations were very low, similar to those presented in the two previous analyses. Many more were not significantly different from zero due to the smaller sample size.

The four factor solutions on Test I for each of the above cases are presented in Tables 47 to 54 in Appendix 5. For Test II the five factor solutions are given in Tables 55 through 62. Many of the patterns described on the previous pages are evident in these tables, however all factors were not interpretable. Due to the low correlations and small portion of accounted variance, no attempt was made to determine if the factor structures were different for the two sexes, or for different ability groups, or for different grade levels.

Question 4

Is the test on sectioning solids useful to the classroom teacher for predicting achievement in geometry?

The analysis for each of the 12 situations described in the previous chapter is reported independently on the following pages. Similarities and differences between the results in the various situations will be discussed in the concluding chapter of this report.

Hypothesis 4 and 4a are restated here for reference since they are tested in most of the situations which follow.

> Hypothesis 4: The efficiency of prediction of geometry performance is not significantly improved by adding tests on sectioning solids to a battery of other student scores including age, sex, the previous year's final mathematics grade, the last mathematics test score and IQ.
>
> Hypothesis 4a: The efficiency of prediction of geometry performance is not significantly improved by adding tests on sectioning solids to the previous year's final mathematics grade <u>or</u> the last mathematics test score <u>or</u> the IQ score.

Situation 5-1

In this grade 5 class, Tests ID, IMC, and I all had significant correlations with geometry achievement. These correlations are reported in Table 22. The best predictor of achievement in this situation was the last mathematics test score with a correlation of .70.

The results of the stepwise regression for the battery of tests are reported in Table 23. The first predictor to enter the regression equation was the last mathematics test

Table 22

Correlation Matrix for Situation 5-1

		1	2	3	4	5	6	7	8	9
1	Sex	1								
2	Age	-.23	1							
3	IQ(T)	-.25	-.10	1						
4	PG*	-.31	-.03	.89	1					
5	LT	-.36	-.12	.56	.54	1				
6	ID	.11	-.21	.16	.18	.48	1			
7	IMC	-.06	-.17	.25	.27	.45	.69	1		
8	I	.03	-.21	.21	.24	.51	.93	.90	1	
9	GEO	-.35	-.01	.50	.63	.70	.50	.49	.54	1

$p(\rho=0) < .01$ if $|r| > .46$

$p(\rho=0) < .05$ if $|r| > .36$

* In the tables and discussion of this section the following abbreviations are used: PG - previous year's mathematics grade, LT - last mathematics test score, ID - Test ID, IMC - Test IMC, I - Test I, GEO - geometry achievement.

Table 23

Stepwise Regression Summary Table for

Situation 5-1 Using All Predictors

Entering Variable	F Value for Entering Variable	Prob. of Entering Variable	R^2 (per cent)	Total F Value
Last Test	27.5	0	49	27.5
Previous Grade	5.61	.025	58	18.8
Test I	3.73	.064	64	15.1

Best Prediction Equation $\hat{Y}_{GEO} = .041 + .39X_{PG} + .54X_{LT}$

score which accounted for 49% of the variance. The second variable to enter the equation significantly was the final mathematics grade in grade 4 which increased the variance accounted for to 58%. Test I was the next to enter the equation, however it did not increase the variance accounted for significantly. Hypothesis 4 was therefore not rejected for situation 5-1.

Using the three sectioning scores with the final mathematics grade from grade 4, the first variable to enter the equation was the previous year's mathematics grade, $F = 18.5$, $p < .001$. Test I was the next variable to enter significantly, $F = 9.90$, $p < .01$ with the percentage of variance accounted for increasing to 60% from 40%. The regression equation containing these two variables was

$$\hat{Y}_{GEO} = 2.2 + .59X_{PG} + 1.3X_{I}.$$

The last mathematics test score accounted for almost 50% of the variance of the geometry test. Neither the addition of Tests ID, IMC, nor I increased this percentage significantly.

Test I correlated .54 with geometry achievement and was a slightly better predictor than IQ which correlated only .50. The addition of IQ to the regression equation containing Test I increased the percentage of variance accounted for from 29% to 45%, $F = 7.70$, $p < .01$. The regression equation was

$$\hat{Y}_{GEO} = -3.2 + .40X_{IQ(T)} + 1.4X_{I}.$$

Hypothesis 4a was therefore rejected for the previous year's final mathematics grade and IQ but not for the last

mathematics test score.

Situation 5-2

For situation 5-2, Test IMC had the highest correlation with geometry achievement of all available predictors. The correlations are presented in Table 24. Hypothesis 4 and 4a were therefore rejected for this situation.

Table 24

Correlation Matrix for Situation 5-2

	1	2	3	4	5	6	7	8	9	10
1 Sex	1									
2 Age	-.28	1								
3 IQ(V)	.10	-.37	1							
4 IQ(NV)	.00	-.28	.67	1						
5 PG	.00	-.12	.35	.42	1					
6 LT	-.01	-.38	.51	.54	.49	1				
7 ID	-.22	.11	.18	.31	.22	.27	1			
8 IMC	-.01	-.04	.28	.42	.30	.16	.68	1		
9 I	-.13	.04	.25	.39	.28	.24	.93	.90	1	
10 GEO	.05	-.12	.41	.49	.43	.45	.41	.58	.53	1

$p(\rho=0) < .01$ if $|r| > .37$
$p(\rho=0) < .05$ if $|r| > .28$

The addition of each of the variables, grade 4 final mathematics grade, the last mathematics test score, and both verbal and nonverbal IQ to the regression equation significantly increased the percentage of variance accounted for by Test IMC. When the grade 4 final mathematics grade

was added, the percentage of variance accounted for increased from 33% to 40%, $F = 5.52$, $p < .05$. For the last mathematics score the increase was to 47%, $F = 10.9$, $p < .01$ and for verbal IQ to 40%, $F = 4.98$, $p < .05$. For nonverbal IQ the increase was to 44%, $F = 5.54$, $p < .05$. In none of the above cases did a third variable add significantly to the prediction. The regression equations for these four situations were as follows.

$$\hat{Y}_{GEO} = 23 + .19X_{PG} + 2.9X_{IMC}$$
$$\hat{Y}_{GEO} = -7.2 + .51X_{LT} + 3.0X_{IMC}$$
$$\hat{Y}_{GEO} = 2.7 + .27X_{IQ(V)} + 2.9X_{IMC}$$
$$\hat{Y}_{GEO} = 5.7 + .26X_{IQ(NV)} + 2.6X_{IMC}$$

In situations which included two or more classes, approximately 25% of the subjects were removed for purposes of cross-validating the regression equations. A cross-validation multiple-R was calculated for each of these situations and compared with the original multiple-R, corrected for shrinkage (Tatsuoka, 1969). In those situations where these two values were similar, the regression equations were considered to be valid.

For situation 5-1, 15 subjects from the original sample were used to validate the above regression equations. After correcting the multiple correlations for shrinkage, the corrected values were compared to each cross-validation multiple-R as follows: .66 to .44 for the equation containing the last mathematics test score, .62 to .57 for the previous year's final mathematics grade, .61 to .57 for verbal IQ and .62 to .59 for nonverbal IQ. In each case the

corrected value was greater than the cross-validation multiple-R, the differences being less than .06 for the latter three equations.

Situation 6-1

The correlations between Tests I, ID, and IMC with geometry achievement were .45, ($p < .01$); .43, ($p < .05$); and .37, ($p < .05$) respectively. The complete correlation matrix is found in Table 25. None of these tests added significantly to the efficiency of prediction provided singly by the last mathematics test score, the previous year's final mathematics grade or IQ. These variables alone accounted for 76%, 74%, and 56% of the variance respectively. The best regression equation obtained from the battery of variables contained the last mathematics test score and the previous year's final mathematics grade, the latter variable increasing the per unit of variance accounted for to 82%, $F = 9.75$, $p < .01$. The equation was

$$\hat{Y}_{GEO} = 13 + .38X_{LT} + .44X_{PG}.$$

Hypothesis 4 and 4a were therefore not rejected for situation 6-1.

Situation 6-2

The correlations for this situation are presented in Table 26. Neither Test I nor Test IMC correlated significantly ($p < .05$) with achievement in geometry. The correlation with Test ID was significant, $p < .05$. Each of

Table 25

Correlation Matrix for Situation 6-1

		1	2	3	4	5	6	7	8	9
1	Sex	1								
2	Age	-.22	1							
3	IQ(T)	-.01	-.22	1						
4	PG	.10	.16	.68	1					
5	LT	.19	-.03	.73	.83	1				
6	ID	.29	-.17	.36	.52	.41	1			
7	IMC	.13	-.06	.38	.48	.36	.57	1		
8	I	.23	-.13	.42	.56	.44	.88	.89	1	
9	GEO	.04	.03	.75	.86	.87	.43	.37	.45	1

$p(\rho = 0) < .01$ if $|r| > .44$

$p(\rho = 0) < .05$ if $|r| > .34$

Table 26

Correlation Matrix for Situation 6-2

		1	2	3	4	5	6	7	8	9	10
1	Sex	1									
2	Age	-.01	1								
3	IQ(V)	-.16	-.47	1							
4	IQ(NV)	-.18	-.50	.69	1						
5	PG	-.04	-.07	.50	.66	1					
6	LT	.32	-.17	.34	.49	.45	1				
7	ID	-.01	-.33	.42	.44	.39	.40	1			
8	IMC	-.13	-.30	.34	.31	.36	.36	.41	1		
9	I	-.08	-.37	.46	.45	.45	.46	.87	.81	1	
10	GEO	.09	-.12	.42	.48	.42	.66	.34	.16	.30	1

$p(\rho = 0) < .01$ if $|r| > .39$

$p(\rho = 0) < .05$ if $|r| > .30$

the other variables, except age and sex, had significant correlations with achievement; however, only the last mathematics test accounted for more than 40% of the variance of the geometry test. In no case did the addition of a second variable to the regression equation increase the multiple correlation significantly. For this situation neither Hypothesis 4 nor 4a was rejected.

The regression equation containing the last mathematics test score was validated using a sample of 14 subjects chosen from the two classes in this situation. The equation was

$$\hat{Y}_{GEO} = 23 + .58X_{LT}.$$

A cross-validation R of .89 was found in comparison with the original correlation of .66 between the last mathematics test and geometry achievement.

Situation 7-1

The correlations for situation 7-1 are found in Table 27. Tests IMC and I both correlated significantly with geometry achievement. The highest correlation with achievement was with the last mathematics test score, $r = .57$. The second variable to load on the regression equation was Test I, significantly increasing the percentage of variance accounted for to 39% from 32%, $F = 4.37$, $p < .05$. The regression equation was

$$\hat{Y}_{GEO} = 17 + .41X_{LT} + 1.4X_{I}.$$

A cross-validation multiple-R for this equation of .63 was

Table 27

Correlation Matrix for Situation 7-1

	1	2	3	4	5	6	7	8	9	10
1 Sex	1									
2 Age	-.01	1								
3 IQ(V)	.18	-.21	1							
4 IQ(NV)	-.05	-.13	.41	1						
5 PG	.05	.06	.63	.41	1					
6 LT	.02	-.05	.33	.39	.48	1				
7 ID	-.09	-.05	.11	.34	.20	.11	1			
8 IMC	-.13	.19	-.03	.46	.24	.35	.54	1		
9 I	-.12	.05	.07	.44	.24	.24	.92	.82	1	
10 GEO	-.17	.02	.31	.39	.44	.57	.30	.42	.39	1

$p(\rho = 0) < .01$ if $|r| > .39$

$p(\rho = 0) < .05$ if $|r| > .30$

found compared to the original multiple-R of .62. Hypothesis 4 was therefore rejected.

The stepwise procedure was conducted using Tests I, ID, and IMC with the grade 6 final mathematics grade, verbal IQ and nonverbal IQ respectively. The addition of Test IMC significantly increased the variance accounted for by the grade 6 final mathematics grade from 19% to 29%, $F = 5.11$, $p < .05$. The regression equation was

$$\hat{Y}_{GEO} = 5.0 + 7.8X_{PG} + 3.5X_{IMC}.$$

The cross-validation multiple-R was found to be .53 compared to the original multiple-R corrected for shrinkage of .50.

Test IMC correlated higher with geometry achievement than either verbal or nonverbal IQ. When considered with

Test IMC, verbal IQ significantly increased the variance accounted for from 16% to 27%, $F = 5.50$, $p < .05$. A cross-validation multiple-R of .61 was found for the regression equation

$$\hat{Y}_{GEO} = -43 + .58X_{IQ(V)} + 4.6X_{IMC}.$$

The addition of nonverbal IQ did not significantly increase the multiple correlation.

Hypothesis 4a was therefore rejected for the previous year's final mathematics grade, the last mathematics test score, and verbal and nonverbal IQ.

Situation 7-2

The final grade 6 mathematics grade had the highest correlation with achievement on Kuper's test, $r = .60$, $p < .01$. The complete correlation matrix is given in Table 28.

Table 28

Correlation Matrix for Situation 7-2 Treatment

		1	2	3	4	5	6	7	8
1	Sex	1							
2	Age	-.43	1						
3	IQ(T)	.20	-.37	1					
4	PG	.05	-.34	.76	1				
5	ID	-.25	.11	.40	.25	1			
6	IMC	-.21	.13	.36	.32	.57	1		
7	I	-.26	.13	.43	.32	.92	.85	1	
8	GEO	-.05	-.25	.45	.60	.32	.41	.40	1

$p(\rho=0) < .01$ if $|r| > .37$
$p(\rho=0) < .05$ if $|r| > .29$

The last mathematics test score was not used as a variable in this situation since classes from three different schools were included in the situation. The previous year's final mathematics grade was retained since in any given classroom, the teacher is faced with students who were in a variety of classes or schools in the previous year.

On the stepwise regression analysis, Test IMC was the second variable to be admitted to the regression equation increasing the variance accounted for by the grade 6 final mathematics grade to 42% from 37%. This increase was not significant, $F = 3.73$, $p = .06$. Therefore, Hypothesis 4 was not rejected.

Test IMC increased the variance accounted for by the IQ score from 20% to 27%, $F = 4.07$, $p < .05$. The regression equation was

$$\hat{Y}_{GEO} = 4.0 + .034X_{IQ(T)} + .31X_{IMC}.$$

The cross-validation multiple-R was .57 compared to an original multiple-R of .52.

Hypothesis 4a was rejected for IQ but not for the previous year's final mathematics grade. It was not tested for the last mathematics test score.

For this situation a control group also took the achievement test administered to the treatment group. The correlations for this group are reported in Table 29. The highest correlation with achievement was with Test ID, $r = .49$, $p < .01$. In no instance did the addition of the other predictors to the regression equation significantly increase the percentage of variance accounted for by this

Table 29

Correlation Matrix for Situation 7-2 Control

		1	2	3	4	5	6	7	8
1	Sex	1							
2	Age	.07	1						
3	IQ(T)	.00	-.24	1					
4	PG	.04	-.16	.78	1				
5	ID	-.12	.07	.42	.43	1			
6	IMD	-.13	-.14	.37	.36	.56	1		
7	I	-.14	-.03	.45	.45	.91	.85	1	
8	GEO	-.14	.14	.40	.33	.49	.29	.45	1

$p(\rho = 0) < .01$ if $|r| > .37$

$p(\rho = 0) < .05$ if $|r| > .29$

test. Both Hypothesis 4 and 4a were rejected since Test ID had the greatest correlation with achievement.

Situation 8-1

The correlations of Test I, ID, and IMC with geometry achievement were .70, .69, and .65 respectively, all significantly different from 0, $p < .01$. The complete table of correlations is found in Table 30.

On the stepwise regression the grade 7 final mathematics grade was the first variable to enter the equation, accounting for 61% of the variance. The second variable to enter the equation was Test I, increasing the variance accounted for to 72%, $F = 13.8$, $p < .01$. The addition of further variables did not significantly increase the

multiple correlation. The regression equation was

$$\hat{Y}_{GEO} = -8.8 + .42X_{PG} + .93X_{I}.$$

The cross-validation multiple-R was .81 compared to a multiple-R corrected for shrinkage of .84. Hypothesis 4 was rejected for this situation.

Table 30

Correlation Matrix for Situation 8-1

	1	2	3	4	5	6	7	8	9	10
1 Sex	1									
2 Age	-.10	1								
3 IQ(V)	-.21	-.33	1							
4 IQ(NV)	-.11	-.18	.63	1						
5 PG	-.27	-.14	.63	.73	1					
6 LT	-.20	-.31	.61	.67	.82	1				
7 ID	-.08	-.36	.45	.58	.52	.60	1			
8 IMC	-.10	-.25	.39	.48	.50	.49	.82	1		
9 I	-.10	-.33	.45	.56	.54	.58	.97	.93	1	
10 GEO	-.24	-.33	.55	.73	.78	.77	.69	.65	.70	1

$p(\rho = 0) < .01$ if $|r| > .41$

$p(\rho = 0) < .05$ if $|r| > .32$

The above indicated that Test I added significantly to the battery of tests, in particular to the final mathematics grade from grade 7. When the last mathematics test score and Tests I, ID, and IMC were subjected to analysis, Test IMC was the second variable to enter the regression equation, significantly increasing the percentage of variance accounted

for by the last mathematics test score from 60% to 70%, $F = 10.8$, $p < .01$. The regression equation was

$$\hat{Y}_{GEO} = 3.6 + .24X_{LT} + 2.0X_{IMC}.$$

The cross-validation multiple-R was .89 compared to the original multiple-R of .83.

All three sectioning tests correlated higher with achievement than verbal IQ. Test I had the highest correlation and entered the regression equation first. The addition of verbal IQ to the equation significantly increased the percentage of variance accounted for from 49% to 56%, $F = 5.39$, $p < .05$. The regression equation was

$$\hat{Y}_{GEO} = -14 + .22X_{IQ(V)} + 1.3X_{I}.$$

The cross-validation multiple-R was .78 compared with an original multiple-R of .75.

The addition of Test I to the equation containing nonverbal IQ increased the variance accounted for to 66% from 53%, $F = 12.4$, $p < .01$. The regression equation was

$$\hat{Y}_{GEO} = -16 + .30X_{IQ(NV)} + 1.0X_{I}.$$

The cross-validation multiple-R was .75 compared to an original multiple-R corrected for shrinkage of .80.

Hypothesis 4a was rejected for each of the previous year's final mathematics grade, last mathematics test score, and nonverbal IQ since the sectioning test increased the efficiency of prediction. It was also rejected for verbal IQ since each of the sectioning tests correlated higher with achievement than did this predictor.

Situation 8-2

This situation, which also was involved in the study by Ong (1976), consisted of two groups. One group received an inventive treatment of motion geometry and the second received a more traditional treatment similar to that of the previous situation. Each of these groups received the same four post treatment measures, two of which were achievement tests and two of which were labeled creative tests. The results of the group receiving the inventive treatment are presented first.

The correlation matrix for the inventive group is presented in Table 31. None of the correlations between Tests I, ID, or IMC with either of the two creative tests, variables 10 and 11, were significantly different from zero. It was therefore concluded that the sectioning tests were not good predictors of achievement on these tests and no further analysis was conducted. Neither Hypothesis 4 nor 4a was rejected for these two variables.

The correlations with the achievement measures, however, were all significant, $p < .01$. First consider the post motion geometry test. On the stepwise regression, the grade 7 final mathematics grade was the first variable to enter the equation. The second variable to enter, Test IMC, significantly increased the percentage of variance accounted for from 58% to 74%, $F = 17.0$, $p < .01$. The addition of further variables did not significantly increase this percentage. The regression equation was

$$\hat{Y}_{GEO} = -9.0 + .30X_{PG} + 1.3X_{IMC}.$$

Table 31

Correlation Matrix for Situation 8-2 - Inventive Group

	1	2	3	4	5	6	7	8	9	10	11	12
1 Sex	1											
2 Age	-.14	1										
3 IQ(T)	.26	-.07	1									
4 PG	.49	.06	.71	1								
5 LT	.32	-.15	.52	.63	1							
6 ID	-.08	.19	.65	.47	.38	1						
7 IMC	-.26	.05	.57	.29	.30	.75	1					
8 I	-.16	.14	.66	.42	.37	.95	.91	1				
9 Post Motion Geometry	.17	.11	.75	.76	.53	.61	.61	.65	1			
10 Creative Motion Geometry	.20	-.13	.35	.40	.30	.19	.30	.25	.41	1		
11 Creative Geometry	.34	-.15	.68	.64	.45	.34	.27	.33	.60	.44	1	
12 Area	.23	-.02	.73	.59	.40	.73	.55	.69	.62	.55	.68	1

$p(\rho = 0) < .01$ if $|r| > .47$

$p(\rho = 0) < .05$ if $|r| > .37$

Hypothesis 4 was therefore rejected for this group on the post motion geometry test.

When the three sectioning tests were considered with the last mathematics test score and the above achievement test, Test I entered the equation first accounting for 42% of the variance. Hypothesis 4a was therefore rejected. The last mathematics test score entered next increasing the variance accounted for to 52%, $F = 5.07$, $p < .05$.

The regression equation was

$$\hat{Y}_{GEO} = 2.1 + .12X_{LT} + .75X_{I}.$$

When IQ was considered with the sectioning tests, it entered the prediction equation first, followed by Test IMC. However, Test IMC did not significantly increase the variance accounted for, $F = 3.26$, $p = .08$. Hypothesis 4a was therefore not rejected for this group with IQ as a predictor of success on the motion geometry test. It was rejected for each of the last mathematics test scores and the previous year's final mathematics grade.

With the second achievement variable, variable number 12, IQ was the first variable to be entered into the prediction equation. Test ID was the next to enter increasing the percent of variance accounted for from 53% to 64%, $F = 7.88$, $p < .01$. The addition of further variables was not significant. The regression equation was

$$\hat{Y}_{AREA} = 28 + .42X_{IQ(T)} + 3.1X_{ID}.$$

Hypothesis 4 was therefore rejected for this variable, as was Hypothesis 4a for IQ as a predictor.

Test ID and Test I correlated higher with achievement on this criterion than either the grade 7 final mathematics grade or the last mathematics test score. Test IMC also correlated higher than did the last mathematics test score. Hypothesis 4a was thus rejected for these variables. When the last mathematics test was added to the regression equation containing Test ID there was no significant improvement in the prediction. When the grade 7 final mathematics grade was added, the percentage of variance

increased significantly from 53% to 61%, $F = 5.27$, $p < .05$. This regression equation was

$$\hat{Y}_{AREA} = 7.7 + .45X_{PG} + 4.1X_{ID}.$$

The correlations for the second group in this situation are found in Table 32. For this group, the correlations between Tests I and ID and variable 10, the creative motion geometry test, were significant at the .05 and .01 level respectively. However, the final mathematics grade from grade 7 was the first variable to enter the regression equation and neither of the above tests added significantly to that prediction. Hypothesis 4 was not rejected nor was Hypothesis 4a for the previous year's final mathematics grade.

Test ID correlated higher with the creative motion geometry test than either of the last mathematics test score or IQ. Hypothesis 4a was rejected for these two predictors. The addition of neither of these variables to the regression equation formed with Test ID significantly increased the efficiency of prediction.

For the second creative test, Tests I and IMC both correlated significantly ($p < .05$) with the test on creative geometry. The correlation in each case was .42. This was the highest correlation of all the predictors. Since the other correlations were not significant, the analysis was not pursued. However, Hypotheses 4 and 4a were rejected in this case.

As with the inventive group, the correlations in this group between the sectioning tests and the achievement tests

Table 32

Correlation Matrix for Situation 8-2 - Regular Group

	1	2	3	4	5	6	7	8	9	10	11	12
1 Sex	1											
2 Age	-.28	1										
3 IQ(T)	.02	-.32	1									
4 PG	.22	-.24	.65	1								
5 LT	.31	.11	.42	.68	1							
6 ID	.15	-.13	.50	.70	.54	1						
7 IMC	.03	-.14	.35	.44	.31	.72	1					
8 I	.05	-.10	.41	.53	.39	.86	.96	1				
9 Post Motion Geometry	.08	-.19	.71	.83	.72	.70	.58	.64	1			
10 Creative Motion Geometry	-.10	.03	.45	.54	.51	.52	.31	.38	.60	1		
11 Creative Geometry	-.09	-.12	.33	.26	.17	.37	.42	.42	.34	.44	1	
12 Area	.07	-.09	.45	.63	.57	.65	.44	.53	.65	.48	.10	1

$p(\rho=0) < .01$ if $|r| > .47$

$p(\rho=0) < .05$ if $|r| > .37$

were much higher than with the creative tests. When variable 9, the post motion geometry test was considered, the four variables - previous year's final mathematics grade, Test IMC, last mathematics test score, and IQ - entered the regression equation significantly in that order. Hypothesis 4 was therefore rejected for this situation. The results of that analysis are presented in Table 33.

Table 33

Stepwise Regression Summary Table for Situation 8-2, Regular Group and Using All Predictors and Post Motion Geometry Test

Variable Entering	F Value of Variable Entering	Prob. of Entering Variable	R^2 (percent)	Total F Value
Previous Grade	62.0	0	70	62.0
Test IMC	5.89	.02	75	39.6
Last Test	4.94	.04	79	32.0
IQ	6.33	.02	84	30.7

Best Prediction Equation

$$\hat{Y}_{GEO} = -2.0 + .15X_{PG} + .56X_{IMC} + .11X_{LT} + .093X_{IQ(T)}$$

The three sectioning tests were then used with each of the grade 7 final mathematics grade, last mathematics test score, and IQ separately. As indicated by the second step in Table 33, Test IMC added significantly to the prediction of post motion geometry scores by the grade 7 final mathematics grade. The regression equation containing these two variables was

$$\hat{Y}_{GEO} = -.93 + .31X_{PG} + .62X_{IMC}.$$

Using the last mathematics test score, Test I increased the variance accounted for by the last mathematics test score from 52% to 67%, $F = 11.9$, $p < .01$. The regression equation was

$$\hat{Y}_{GEO} = 2.3 + .21X_{LT} + .47X_{I}.$$

Test ID was the second variable to enter the equation when IQ was used, raising the percentage of variance accounted for to 67% from 50%, $F = 12.5$, $p < .01$. The regression equation was

$$\hat{Y}_{GEO} = 9.5 + .16X_{IQ(T)} + 1.0X_{ID}.$$

Hypothesis 4a was therefore rejected for each of these predictor variables.

The second achievement variable, a test on area, correlated highest with Test ID, $r = .65$, $p < .01$. None of the remaining predictors, when added to the regression equation containing this test, significantly improved the variance accounted for. The regression equation was

$$\hat{Y}_{AREA} = 28 + 3.2X_{ID}.$$

Therefore Hypotheses 4 and 4a were rejected for the test on area.

Situation 9-1

The correlations of Tests I, ID, and IMC with geometry achievement in situation 9-1 were .28, .24, and .26 respectively. None of these correlations were significantly different from zero. The grade 8 final mathematics grade, the last mathematics test score, and verbal IQ all had significant correlations with the criteria, .73, .58, and .63 respectively. The complete correlation matrix is found in Table 34. Due to the nonsignificant correlations further analysis was not pursued and it was concluded that Hypotheses 4 and 4a were not rejected.

Table 34

Correlation Matrix for Situation 9-1

	1	2	3	4	5	6	7	8	9	10
1 Sex	1									
2 Age	.07	1								
3 IQ(V)	-.21	-.44	1							
4 IQ(NV)	-.22	-.52	.52	1						
5 PG	.07	-.24	.42	.70	1					
6 LT	-.08	-.21	.37	.57	.82	1				
7 ID	.07	-.07	.29	.08	.32	.42	1			
8 IMC	.11	-.34	.19	.22	.32	.45	.58	1		
9 I	.11	-.23	.27	.17	.36	.49	.88	.89	1	
10 GEO	.05	-.35	.28	.63	.73	.58	.24	.26	.28	1

$p(\rho = 0) < .01$ if $|r| > .40$

$p(\rho = 0) < .05$ if $|r| > .31$

Situation 9-2

The correlation matrix for this situation is found in Table 35. The correlation of Test ID with geometry achievement was not significant whereas the correlations between Tests I and IMC, and geometry achievement were .30 and .31 respectively, significant at the .05 level.

Using the stepwise regression procedure the variables entered the equation in the following order: the previous year's final mathematics grade, $F = 228$, $p < .01$; last mathematics test score, $F = 8.16$, $p < .01$; and sex, $F = 4.68$, $p < .05$. No other variables added to the equation increased the multiple correlation significantly. The regression

equation was

$$\hat{Y}_{GEO} = 10 + .62X_{PG} + .29X_{LT} - 4.2X_{Sex}.$$

The cross-validation multiple-R for this equation was .94 compared to an original multiple-R of .91. Hypothesis 4 was not rejected since the addition of none of the sectioning tests increased the multiple correlation significantly.

Table 35

Correlation Matrix for Situation 9-2

		1	2	3	4	5	6	7	8	9
1	Sex	1								
2	Age	.03	1							
3	IQ(T)	-.06	-.19	1						
4	PG	.08	-.07	.55	1					
5	LT	.07	-.02	.50	.91	1				
6	ID	-.34	-.07	.44	.18	.21	1			
7	IMC	-.23	-.28	.53	.26	.27	.66	1		
8	I	-.31	-.19	.53	.24	.27	.91	.91	1	
9	GEO	-.05	-.07	.52	.89	.87	.23	.31	.30	1

$p(\rho=0) < .01$ if $|r| > .31$

$p(\rho=0) < .05$ if $|r| > .24$

The last mathematics test score, the previous year's final mathematics grade, and IQ accounted individually for 78%, 76%, and 27% of the variance of the geometry criterion. In no instance did the addition of the sectioning tests significantly increase these percentages. Hypothesis 4a was therefore not rejected for any of these predictors.

Situation 10-1

The correlations shown in Table 36 show significant correlations, $p < .05$, for Tests I, IMC, and ID with geometry achievement. Verbal IQ was the best predictor accounting for 40% of the variance of the criterion. The second variable to enter the regression equation, the final grade 9 mathematics grade, significantly increased the percentage of variance accounted for to 55%, $F = 10.9$, $p < .01$. The further addition of variables did not significantly increase this percentage. The regression equation was

$$\hat{Y}_{GEO} = -34 + .57X_{IQ(V)} + 8.3X_{PG}.$$

Hypothesis 4 was not rejected for this situation.

Table 36

Correlation Matrix for Situation 10-1

	1	2	3	4	5	6	7	8	9	10
1 Sex	1									
2 Age	-.12	1								
3 IQ(V)	.01	-.41	1							
4 IQ(NV)	.25	-.35	.45	1						
5 PG	.01	.10	.39	.36	1					
6 LT	.13	-.20	.38	.28	.60	1				
7 ID	.14	-.06	.18	.48	.27	.32	1			
8 IMC	-.16	-.01	.18	.25	.39	.43	.57	1		
9 I	-.01	-.04	.20	.41	.38	.42	.88	.89	1	
10 GEO	-.07	-.09	.63	.43	.61	.60	.36	.35	.40	1

$p(\rho=0) < .01$ if $|r| > .43$

$p(\rho=0) < .05$ if $|r| > .33$

The addition of the sectioning tests to the last mathematics test score, the previous year's final mathematics grade, or nonverbal IQ did not significantly add to the prediction equation. The addition of Test I to verbal IQ increased the percentage of variance accounted for from 40% to 47%, $F = 4.46$, $p < .05$. The regression equation was

$$\hat{Y}_{GEO} = -40 + .70X_{IQ(V)} + 1.1X_{I}.$$

Hypothesis 4a was therefore rejected for verbal IQ but not for the previous year's final mathematics grade, the last mathematics test score, or nonverbal IQ.

Situation 10-2

The correlation matrix for this situation is found in Table 37. The only predictor correlating significantly with the geometry criterion was the last mathematics test score. This correlation was .39 and was significant at the .05 level. Further analysis was not pursued. Hypotheses 4 and 4a were not rejected for situation 10-2.

Summary

The results of the previous 12 situations are summarized in Table 38. In those cases where it is indicated that the hypothesis was rejected, either one of the sectioning tests correlated higher with the geometry criterion than any of the other predictors concerned, or significantly entered a regression equation already containing one or more of these other predictors. In instances where the hypothesis was not rejected, either

Table 37

Correlation Matrix for Situation 10-2

	1	2	3	4	5	6	7	8	9	10
1 Sex	1									
2 Age	-.14	1								
3 IQ(V)	-.17	.01	1							
4 IQ(NV)	-.21	-.19	.28	1						
5 PG	.04	-.10	-.27	-.25	1					
6 LT	-.16	.13	.36	-.01	.18	1				
7 ID	.04	-.04	.44	.35	-.24	.33	1			
8 IMC	-.07	.39	.10	-.12	-.15	.32	.05	1		
9 I	-.01	.16	.42	.23	-.28	.44	.87	.54	1	
10 GEO	-.11	-.08	.21	.17	.11	.39	.22	.09	.23	1

$p(\rho = 0) < .01$ if $|r| > .47$

$p(\rho = 0) < .05$ if $|r| > .37$

the correlations of the sectioning tests with the criterion were not significantly different from zero, or they did not increase the percentage of variance of the criterion accounted for when added to a regression equation containing the other predictors. These results are further discussed in the concluding chapter.

Table 38

Summary of Results of Question 4

Situation	Hypothesis 4	Hypothesis 4a				
		PG	LT	IQ(V)	IQ(NV)	IQ(T)
5-1	N*	Y**	N	-	-	Y
5-2	Y	Y	Y	Y	Y	-
6-1	N	N	N	-	-	N
6-2	N	N	N	N	N	-
7-1	Y	Y	Y	Y	Y	-
7-2						
Treatment	N	N	-	-	-	Y
Control	Y	Y	-	-	-	Y
8-1	Y	Y	Y	Y	Y	-
8-2 Inventive						
Post motion geo.	Y	Y	Y	-	-	N
Creative motion geo.	N	N	N	-	-	N
Creative geo.	N	N	N	-	-	N
Area	Y	Y	Y	-	-	Y
8-2 Regular						
Post motion geo.	Y	Y	Y	-	-	Y
Creative motion geo.	N	N	Y	-	-	Y
Creative geo.	Y	Y	Y	-	-	Y
Area	Y	Y	Y	-	-	Y
9-1	N	N	N	N	N	-
9-2	N	N	N	-	-	N
10-1	N	N	N	Y	N	-
10-2	N	N	N	N	N	-

* Hypothesis not rejected

** Hypothesis rejected

CHAPTER VI

Summary and Conclusions

In this final chapter, a summary of the investigation is presented. The results reported in the previous chapter are discussed and interpreted. Finally, some implications for teachers and for future research are outlined.

Summary of the Study

Two tests on sectioning solid figures were designed for use in the study. Students were required to identify 16 different geometric sections resulting from four different cuts on each of four different solids on each test. Each test consisted of two subtests. On one the respondents were required to draw freehand their representation of each section, and on the second they had to select the correct representation from a set of distractors. The solids used on the first test, Test I, were the cube, the triangular prism, the parallelepiped, and the cone. The Test II solids were the rectangular prism, the cylinder, the four pointed star, and the square pyramid. The four cuts were labeled as longitudinal, transverse, parallel, and oblique. Each was defined precisely as a function of the direction of movement of the cutting instrument, the orientation of the solid, and the position of the cut on the solid.

Test I was administered to a sample of 38 intact

classes of students from the Edmonton Public and Separate School systems. The classes were representative of grades 5 to 10 in the city of Edmonton. The test was given to each class just prior to their regularly scheduled unit on geometry. Since geometry was taught at different times in different classes the testing was conducted over a 7 month period.

The unit on geometry was taught by the regular mathematics teacher who made all decisions with regard to what was taught, how it was taught, and the methods of evaluation employed. The duration of the unit also depended on the particular situation. In short, the existing ecology of the classroom was not disturbed except to administer the sectioning tests.

After the unit on geometry had concluded the researcher administered Test II to each class. Data collected on each student, in addition to the results on Tests I and II, included age, sex, the previous year's final mathematics grade, the last mathematics test score, and IQ. These data were readily available to the classroom teacher in each instance.

The first purpose of the study was to provide an indepth analysis of the sectioning tests. To fulfil this purpose 432 subjects were randomly selected from those tested, 12 occupying each cell of a 6 x 2 x 3, grade by sex by ability design. The grades were 5 through 10 inclusive and the ability levels were determined by Lorge-Thorndike IQ scores. Of all subjects tested, approximately

equal numbers were classified as being high, average, and low in ability based on these scores.

The analysis of the data included a descriptive treatment of the performances on each sectioning item; a multivariate analysis of variance to investigate differences due to grade, sex and ability; and a principal axis factor analysis to determine if the ability of sectioning solids was a uni-factor trait or perhaps a composite of several independent or related abilities. This factor analysis was exploratory in nature.

The second purpose of the study was to investigate the utility of the sectioning tests to the classroom teacher in predicting achievement in geometry. Individual classes or groups of classes receiving similar geometry treatments were selected, and analysis carried out to determine the predictive validity of Test I and its subtests for each situation. Two situations at each grade level were selected for this purpose.

Discussion of the Results

Prior to discussing the results presented in the previous chapter, two observations based on those results are noted and discussed. These observations arise from the fact that for each grade level, except grade 5; each sex; each ability level; and for the sample as a whole, the test means increased from Test ID to IMC to IID to IIMC. This increase paralleled the order of administration of the tests.

First, total individual scores on Test II were consistently higher than those on Test I, the means being higher in each case mentioned above. The significance of these differences was not tested. The selection of solids for the two tests was based on Bober's (1973) claim that the two sets were of equivalent difficulty. Possible explanations for the apparent inconsistency between Bober's claim and the results of this study are as follows.

The various geometry treatments between the two tests in this study could have affected the performance on Test II. The increase in scores occurred in every class used in the study, no matter what the treatment or the duration. If the ability to section solids was affected by the various geometry treatments, it appeared that the affect was similar in each case. Bober (1973) found that subjects who were exposed to "rich experiences in projective and Euclidean geometry" increased in their ability to section solids more than did those not receiving these experiences. The geometry treatments of this study were much different from those used by Bober. They did not, for the most part, include active manipulation of concrete materials, and they were not closely related to sectioning experiences as were Bober's. It was felt that any treatment effect in this study was consistent across treatments and was minimal.

The shortness of the units would appear to discount any theory that maturation would account for the increase in test scores, except perhaps in individual instances due to quantum jumps. These jumps, according to the Van Hiele

theory, occur when the pupil seems to have "matured" and moves to the next level of geometric thought (Wirszup, 1976).

Test I might have served as training for Test II, however the time lag of 4 to 7 weeks, although short, was long enough that this effect should have been minimal. The results of Test I were not returned to the students until after Test II had been administered, therefore the students had no formal indication whether or not they had been successful on the first test.

A final explanation could have been that Bober's claim was false and that in fact Test I was more difficult than Test II. Test I contained three sections which were very difficult for students from every strata of the sample. These three sections alone, the oblique section on the cube and triangular prism, and the transverse section on the cone, accounted for most of the difference in the means of the two tests. Test II did not appear to have sections of corresponding difficulty. For this reason, Bober's claim is disputed and it is assumed in the remainder of this discussion that overall, Test I was more difficult than was Test II.

The second observation arising from the means was that the total scores on the multiple choice subtests of Tests I and II were greater than the corresponding scores on the drawing subtests. One explanation for this result could have been the effect of always administering the drawing subtest prior to the multiple choice subtest. Since the subjects were viewing the sectioning tasks for the second

time in the multiple choice format, it might be expected that they would do better. However when individual sections were considered, there were instances where scores on the multiple choice subtest were lower than corresponding scores on the drawings. For some sections the multiple choice distractors appeared to suggest plausible answers not considered when the subject was using the drawing method of response.

The second explanation could be that it was more difficult to represent the sections by drawings than by selecting them from sets of distractors. This disputes the claim by Piaget and Inhelder (1967) that the two modes of response are equivalent. Again the answer appeared to depend on the particular solid. Some sections were very difficult to represent by drawing, the difficulty depending upon the section under consideration. The results of the previous chapter strongly support this statement. Which mode of response was more difficult depended on the particular section involved. This conclusion is also in conflict with that of Piaget, however it is supported by Boe (1966) who stated that the two modes of response need not measure the same thing.

To summarize the above, the assumptions made for the purposes of the ensuing discussion were that Test I, as a whole, was more difficult than Test II, and that each drawing subtest was more difficult than the corresponding multiple choice subtest. However, for some sections the drawing mode of response proved to be easier than the multiple choice mode. The difficulty of any particular item depended on

proved to be an easier method of representation, whereas with others the multiple choice mode was easier. These points are elaborated upon in the following pages.

Purpose 1 Test Results

The first question that this study attempted to answer was how do students respond to sectioning tasks involving various cuts and solids? Piaget and Inhelder (1967) claimed that the ability to section solids is fully developed in most youngsters by age 12. The average age of the grade 7 students in the present sample was 12 years 7 months. With the exception of the oblique section on the cube, parallelepiped, cone, pyramid, and the triangular and rectangular prisms; and the transverse section on the cone, each section was correctly represented by over 70% of these grade 7 students using either the drawing or multiple choice mode of response or both. For the remaining sections even grade 10 students experienced considerable difficulty in most cases.

The contention that 12 year olds have mastered the sectioning tasks is supported by the results of this study for those sections not resulting from oblique cuts, or from the transverse section on the cone. The difficulty with oblique sections is consistent with the results of previous sectioning experiments (Boe, 1966; Davis, 1969; Pothier, 1975), each of which indicated that the oblique cut was "the most difficult". Difficulties that preschool and early elementary school children experience with obliqueness and

diagonality have been well documented (Olson, 1970; Bryant, 1974). The difficulty with this orientation, at least on sectioning tasks, appears to extend well into the adolescent years.

As suggested previously in this report, three of the sections in Test I were of "extreme" difficulty for almost all subjects. These were the oblique section on the cube and triangular prism, and the transverse section on the cone. Davis (1973) remarked that the oblique section on the cube was somewhat misleading due to the resultant rectangular section and all the square faces. The same could be said for the triangular prism where a slight rotation changed the oblique section from an isosceles triangle with a vertical angle which must be greater than 60° to one which must be less than 60°. The curvature of the transverse section on the cone posed problems for many subjects.

One possible explanation for the lack of success on these items lies in the Van Hiele theory (Wirzup, 1976). The fourth level of geometric thought proposed in the theory incorporated the understanding of the significance of deduction whereas in the third level, deduction appeared only in conjunction with experimentation. Perhaps to solve the more difficult sectioning tasks mentioned above, students need to be operating at level 4 to be successful. Or perhaps those operating at level 3 could only experience success if they were allowed to manipulate the solids themselves in a concrete fashion. They were not allowed

this active manipulation in this study.

Two sections which were very easy for students at all grade levels were the parallel section on the cone and cylinder. Both sections were circles.

For several sections a smaller proportion of students at each grade level was successful using the drawing mode of response than with the multiple choice mode. This suggests that these sections were difficult to represent by a drawing. This phenomenon occurred on both Tests I and II and the reader is referred back to Tables 2 and 3 in the previous chapter for evidence of this. Sections exhibiting this property appeared to be of three types. The remarks which follow concerning errors made in the drawings are based on the author's experience in scoring the drawings. A formal system was not devised for classification of the errors.

First, all sections which were equilateral were more difficult to draw than to select from distractors. These included squares, the equilateral triangle, and the star shaped section. The drawings contained many errors in the length of the segments. The squares were often represented as rectangles, the equilateral triangle as isosceles or scalene, and the star shape was composed of isosceles rather than equilateral triangles. This "failure to conserve length" was also observed in a study by Kidder (1976) on the ability of 9, 11, and 13 year olds to represent transformations. Students appear to concentrate on properties of the figures other than the congruence of all sides.

The second type of section which was difficult to represent by drawing was the parallelogram. Here, the most common error was the failure to represent opposite sides with lines of the same slope. This suggests difficulty with the representation of congruent (or perhaps supplementary) angles. Piaget, et al. (1960) indicated that children experience difficulty with the construction of angles until the age of 11. It is indicated by the results of this study that this difficulty may extend well into the junior high school years.

The drawing of the ellipse also posed difficulty for many subjects. As was the case with the hyperbola discussed earlier, difficulties were encountered with the curvature of the figure. The sides were often represented as straight lines or in some instances the curvature was convex rather than concave.

The ability to represent these figures by drawing may require students to operate deductively, perhaps at the Van Hiele level 3. Many of the younger students in the sample could have been operating at only level 2. At this level figures are recognized by their properties but the properties are not connected with one another. For the sections discussed above, perhaps the students focused on only some of the properties and ignored the others. Since they were unable to deduce the remaining properties their representations were incorrect. In the multiple choice format these additional properties might have been recalled by observing the distractors.

Contrasted with the above sections are those which were rectangles or isosceles triangles. With these sections, the proportion of students at each grade level answering the items correctly in the multiple choice mode was similar to or less than the proportion answering the same items in the drawing mode. The sets of distractors for these items might have suggested possibilities that the students had not previously considered. For example, a rectangle shape with curved ends was often selected as the longitudinal or transverse section on the cylinder. Yet this feature was incorporated in only a few drawings of these sections.

In addition to the method of response and the resultant section, two other factors could have influenced the difficulty of a given sectioning task - the cut and the solid. The results of the study showed a dependence existed between these two factors. Previous studies which used the cube, rectangular prism, cylinder, and cone (Boe, 1966; Davis, 1969; Pothier, 1975) have all labeled the oblique cut on these solids as "the most difficult cut" and the cone as "the most difficult solid". Statements such as these must be interpreted with caution due to the dependence which exists between the cuts and solids.

The contention that the oblique cut was more difficult than the other cuts was supported by the results of this study. Yet on the cone, the transverse cut was more difficult under both modes of response; on the star the transverse cut was again more difficult to draw; and on the cylinder the longitudinal cut was more difficult to

select from the distractors. In these selected instances the oblique cut was therefore not "the most difficult".

Of the solids used on the two tests, none emerged as being consistently more difficult than the others. The fewest number of total correct responses occurred with the cone, as in the previous studies; however, the oblique and transverse sections on this solid were among the most difficult of all sections whereas the longitudinal and parallel sections were among the easiest.

The difficulty of a particular section depended on both the solid and cut involved; however, the relationship was by no means consistent. For example, as indicated above the parallel section on the cone was much easier than the transverse section, yet on the cube fewer students represented the parallel section correctly than the transverse section. Dodwell (1963) earlier indicated very little consistency in performance from one sectioning task to another.

In summary the difficulty of a particular section cannot be inferred only from the properties of the cut and solid involved. The difficulty is unique to the given situation. Also the type of resultant section and the mode of response are important factors. Some sections were easier to draw, others easier to select from the distractors.

Grade level, sex, and ability level were all found to be significant factors in the ability to section solids. Although each of these main effects was statistically significant, evidence of significant interactions among the

variables was also found. These results are discussed and interpreted below.

Males consistently scored higher than females on all tests at each grade level and each ability level. Differences between the sexes at each grade level were similar. However, when ability was considered the differences were much greater between males and females of low ability than between those of average and high ability. In particular, females of low ability experienced considerable difficulty with the sectioning tasks. Boe (1966), Davis (1969), and Pothier (1975) all reported males scoring higher than females. Bober (1973) did not find significant differences between the sexes and Piaget and Inhelder (1967) did not examine sex differences. No significant interactions between sex and grade or ability level were reported in the above studies.

High ability students consistently scored higher than those of average ability who in turn scored higher than low ability students. Significant ability effects were also reported by Boe (1966) and Davis (1969). These results held for each sex and for every grade level except grade 5. The grade 5 students of average ability scored lower than those of low ability. The greatest increase between consecutive grade levels for average ability students occurred between grades 5 and 6. Perhaps a significant increase in sectioning ability occurs in average ability students at that level. The Van Hiele theory suggests learning occurs in quantum leaps. A quantum leap from one Van Hiele level

to the next higher one may have occurred here. A similar leap appeared to occur between grades 8 and 9 with low ability students. Perhaps had grades 3 and 4 been included in the sample, this phenomenon might also have been noted with high ability subjects.

The ability to section solids increased at each grade level in the sample with the exception of a slight decline from grade 9 to grade 10 in most instances. The differences between scores in grades 9 and 10 were small. One explanation for this could be a ceiling effect on the test scores. This same effect would account for the very little change in performance of high ability students from grade 8 to grade 10.

Grade 9 and 10 students scored significantly higher than grade 5 and 6 students in the sample. The grade 7 and 8 scores fell between these two extremes. This pattern is somewhat different from that of previous studies. Boe (1966) found no significant differences between grades 8, 10 and 12. Davis (1969) found both grades 8 and 10 scored higher than grade 6, however there were no significant differences between grade 8 and 10 students. Pothier (1975) reported grade 10 students to be significantly better than grade 8 and grade 6 students; however, the differences between grades 6 and 8 were not significant. Bober (1973) found his grade 9 subjects to score significantly higher than either his grade 7 or 8 subjects, but reported no significant difference between grades 7 and 8. These results suggest a significant increase in the development of

sectioning ability between grade 6 and grades 9 and 10 with perhaps a major change between grades 8 and 9.

It has been previously noted that the majority of the tasks, with the exception of the oblique tasks, were answered correctly by most of the subjects by grade 7. Much of the increase in scores then was due to a better performance on the oblique sections. Still it is recalled that even some of these sections were very difficult for students in grades 9 and 10. Recognition of the oblique sections on many solids may require that students operate at the Van Hiele level 4. At this level they must not only use deduction, but also understand the process. Prior to grade 9 very few students could operate at this level.

Additional solids and modification in the cuts from those used in the previous studies were incorporated in this study. The class-group method of testing was different from the individual testing methods of Piaget and Inhelder (1967) and Boe (1966) and from the small group procedures used by Davis (1969). Despite these variations in design, the results from the present study remain very much the same as those of previous studies. Grade level (or age), sex, and ability all are significant factors in the ability to section solids.

It has been suggested earlier in this chapter that the difficulty of a particular section depends on several factors - the type of cut, the solid, the mode of response and the resultant section. Yet even by knowing these factors, it appears one cannot successfully predict the

difficulty in performing a particular sectioning task. The correlations between items on each of the tests were consistently low. The highest of these correlations occurred between items involving the same section under the two modes of response. However, even these correlations accounted for only a small portion of the common variance between the two items.

The results of the factor analysis based on these correlations revealed very few conclusions which can be stated with certainty. The approach taken here is to discuss the results from two points of view - first to mention trends that appeared to exist in the factor solutions, and secondly, to suggest trends which did not appear in the solutions.

The most notable feature of the factor solutions was the tendency for items involving the same sectioning task under both modes of testing to load on the same factor. This lends support to the claim by Piaget and Inhelder (1967) that the drawing and multiple choice modes of response are equivalent. Despite these loadings it must be recognized that the original correlations between the two items were low.

The second feature noted was the tendency for sections of the same type to load on the same factor. For example, sections which were rectangles tended to load together, as did the trapezoids and parallelograms. The ellipse and hyperbola also often appeared on the same factor. This suggests that the type of section may be an important

determinant in the child's ability to section solids.

Finally, several cuts on the same solid tended to load together. All four sections on the parallelepiped, all parallelograms, often appeared on the same factor, as did the three rectangular sections on the star. The longitudinal and transverse sections on the triangular prism consistently had high loadings on the same factor for Test I, as did the same sections on the cylinder for Test II. These results show that at least some significant relationships existed among the sectioning tasks and that each task did not represent a unique factor.

More interesting are the factors which did not emerge from the analyses. Prior to the study it was felt that drawing ability would be a factor on the sectioning tests. However, the factor structures indicated very little evidence that the two modes of response required different abilities. In fact, as suggested above, the tendency of the two items on the same section to load together supports the opposite point of view.

Also, prior to the study, it was felt that a factor would emerge based on the oblique cut. There was little evidence that such a factor existed. Nor was there evidence that any of the three other cuts consistently loaded together on the same factor. Similar conclusions could be stated for the solids. In the cases mentioned above where solids such as the parallelepiped or star might have determined a factor, the shape of the sections, parallelograms and rectangles respectively, might have been the more

dominant force.

The above observations were made, based not only on the factor structures obtained from the entire sample, but also from those obtained for each sex, each ability level, and the grade levels. The factor structures obtained from each of these groups were not easily interpretable and in some cases may have been different from one another. However, each structure appeared to contain one or more of the elements discussed above. Because the ability to section solids as measured on the tests as a whole differed between subjects at different grade levels, of different sex, and of different cognitive ability, one might expect the factor structures to differ. The results of this study are far from conclusive leaving the area ripe for further investigation. Different abilities appear to be necessary to perform different sectioning tasks. Some tasks may even require an ability unique to that task. Other groups of tasks possibly require similar abilities.

Purpose 2 Prediction Results

This study was conducted in a manner which would lead to easy applicability in the classroom. The tests were administered to classroom groups. Despite this fact, the results closely parallel those of studies using individual or small group procedures. Thus the utility of the test is not affected by the administration procedure and the current sectioning test could be used in the classroom.

In what way might the sectioning test prove useful?

The results from the various administrative settings which were studied are discussed below. That the sectioning tests were predictively useful, particularly at grades 5 and 8 is established by the results summarized in Table 38. The how and why of this utility can only be seen by considering particular situations.

In each situation where two or more classes were investigated, a cross-validation of the prediction equations was carried out. In almost all cases the cross-validation multiple-R was very close to the original multiple-R. This gives some evidence of the validity of the prediction equations, using sectioning test scores as predictors of geometry achievement, at least for those situations tested. The following discussion further examines these situations.

Because of content and methodological similarities, the four grade 5 and 6 settings will be considered together. In grade 5 the emphasis was on introductory work with angles, constructions, and perimeter, area, and volume. The grade 6 content included a review and extension of these concepts with more emphasis on the terminology, formulae, and point-set approach to geometry. The teaching techniques were a combination of demonstration, lecture, and discussion with emphasis placed on providing each student with adequate practice activities.

Despite the similarities in the situations at the two grade levels, the results were quite different. At the grade 5 level, the correlations between the sectioning tests and geometry achievement were significant whereas the results

for grade 6 included lower correlations, some of which were essentially zero. As a result the sectioning tests made significant contributions to predicting geometry achievement in grade 5 but not in grade 6.

Several factors could account for these differences. In grade 5 many of the concepts were introduced for the first time. In grade 6 these same concepts were reviewed and extended. Perhaps the sectioning tests predict more efficiently in the earlier part of a presentation of a particular strand of content. Also the grade 6 treatment was more formal from a mathematical point of view. More emphasis was placed on terminology, formulae, and the point-set approach. Perhaps the sectioning tests are more useful in situations where the geometry is more intuitive and informal. A final reason might be due to difference in performance in the sectioning tests between students at these grade levels. It is recalled that there was a sharp increase in the scores of average students between grades 5 and 6. This could lead to differences in the efficiency of prediction of achievement.

The treatment in situation 7-2 consisted of activities which required the students to build rectangular boxes, manipulate them and hence discover properties of their nets, volumes, and diagonals (Kuper, 1975). Prior to the study, it was expected that the results of the sectioning tests would correlate highly with performance on this unit. For the treatment group, correlations between the sectioning tests and achievement were low. Even when considered with

the other predictors, only a small portion of the variance of the achievement test was accounted for. For the control group which received the same test but no treatment, the drawing subtest was the best predictor of achievement. However, the addition of other predictors did not increase the efficiency of prediction of this subtest, the total variance accounted for amounting to only 25%.

The treatment in the other grade 7 situation consisted of the study of the classification and congruence of polygons and circles, and an introduction to the basic concepts of motion geometry - slides, flips and turns. The correlations between the sectioning tasks and achievement were also low, similar to those in situation 7-2 discussed above. The addition of a sectioning test score to the regression equations containing each of the other predictors significantly increased the efficiency of prediction. As in situation 7-2 the proportion of the variance of the achievement scores accounted for was low, in the order of 25% to 40%. For the grade 7 situations in this study the sectioning tests were as useful as the other predictors examined. However, even when considered in conjunction with these other predictors, the sectioning tests appear to be of limited use in predicting geometry achievement at this level.

The highest correlations between the sectioning tests and geometry achievement were found in the grade 8 situations. It was in these same situations that the sectioning tests made their greatest contributions to the

prediction equations. The geometry content in these situations focused on the study and use of motion geometry concepts. The teaching techniques in situation 8-1 and the regular group of situation 8-2 were similar and consisted of the discussion of concepts followed by practice. In the inventive group of situation 8-2, the technique of asking and encouraging open-ended questions was the dominant feature. Regardless of the treatment, the sectioning tests contributed significantly to the prediction of achievement in geometry in grade 8.

The sectioning tasks required the students not only to envision the section, but also to transform it from the solid to their own drawing or to a selection of drawings. It might therefore be expected to find high correlations between this type of transformation activity and the performance on the motions (i.e., transformations) contained in the curriculum.

In situation 8-2, Ong (1976) administered two "creative" tests as well as the achievement tests. These tests contained open-ended questions where the student gave as many different answers as he or she wished, whereas the achievement tests required a single answer to a multiple choice question. In terms of task style, one would expect a relationship between the sectioning tests and the creativity tests. This was not the case. The sectioning tests were poor predictors of achievement on these "creative" tests with nonsignificant correlations in several instances. It may be that the different results for the achievement

and "creative" tests could be due to the type of response required. The sectioning tests might better predict success on tests requiring convergent responses than on those requiring divergent answers, or it might be that the nonoblique items on the sectioning tests formed a convergent task for these students. There might indeed be a relationship between some subset of the sectioning tasks and creative items.

The correlations between the sectioning tests and geometry achievement were very low in both grades 9 and 10. This may have been due to the lower variance of scores on the sectioning tests at these grade levels. Most of these students could successfully represent most of the sections, and those sections that presented difficulty were difficult for almost everybody. The approach taken to geometry in these grades was very formal. Students calculated surface areas and volumes by substitution into formulae or proved geometric deductions. Very little attention was given to spatial activities. The sectioning tasks did not make significant contributions to prediction of geometry achievement at these levels.

A main question in the study was "Are the sectioning tests useful?" From the above we can answer yes with certainty for some situations. How and why they are useful are more complicated questions. One of the possible reasons given for the ability of the sectioning tests to predict achievement in grade 8 was the content - motion geometry. Yet in the grade 7 situation where the content was also

motion geometry, correlations between the sectioning tests and geometry achievement were much lower. Why? It was also suggested that the sectioning tests may have predicted achievement better in grade 5 than in grade 6 since in the former grade the content was introductory, whereas in the latter it was review and extension. In grade 8 much of the content was review and extension of concepts introduced in grade 7, yet the sectioning tests were better predictors of achievement in grade 8. Why? These questions can only be answered by studies carefully designed for that purpose.

The sectioning tests predicted achievement in geometry best at the grade 5 and 8 levels. As discussed earlier the ability to section solids appears to reach plateaus around grades 6 to 7 and again around grades 9 to 10. Perhaps it is during the periods just prior to these plateaus for example, grades 5 and 8, where the ability to section solids is developing in many students that the sectioning tests can be most useful to the geometry teacher. Again the concept of the Van Hiele stages might be relevant here. At these points in time perhaps many students are just advancing from one stage to the next higher stage. At grade 5 many students may be operating at the Van Hiele level 2 and are just beginning to reach level 3 where properties are seen to follow from one another logically. At grade 8 many students are on the verge of entering level 4 where they understand the process of deduction. Again these conjectures need further study.

Previous research has indicated that a student's

performance on mathematics in the immediate past is a good predictor of achievement in geometry (Hanna, 1966). In the present study, the previous year's final mathematics grade and the last mathematics test score consistently correlated highly with geometry achievement. Yet in many instances mentioned above the sectioning tests correlated higher with achievement than did these predictors, or in other cases increased the efficiency of prediction when considered in conjunction with them. The utility of the sectioning tests to the classroom teacher, particularly at the grade 5 and 8 levels has been shown with reasonable certainty.

Summary of the Results

The first two questions of this study were designed to provide further evidence on how students respond to sectioning tasks involving particular solids and cuts, and on the effects of grade, sex, and ability on sectioning solids. The results of the study with respect to these questions are stated below. The final two questions were exploratory in nature and were designed to provide information about the factorial construction of sectioning ability and the utility of the sectioning tests in predicting geometry achievement. The results related to these questions are much more tentative and require further investigation. The first eight results relate to the first two questions, the remainder to the last two.

1. By grade 7 most students could successfully respond to sectioning tasks involving the longitudinal, transverse,

and parallel cuts. The one exception to this was the transverse section on the cone.

2. Sections resulting from the oblique cut were consistently more difficult to represent than those resulting from other cuts. For some oblique cuts even grade 9 and 10 students had only limited success. This also was the case for the transverse cut on the cone.

3. The difficulty of a particular section was independent of the particular cut and solid, except as noted above for the oblique cut.

4. Some sections were easier to select from distractors than to draw. These included sections which were equilateral, parallelograms, ellipses, and the hyperbola.

5. The ability to section solids increased with grade level. Grade 9 and 10 students were significantly better than grades 5 and 6 with a period of development apparently occurring between grades 6 and 9.

6. Males had greater ability to section solids than did females. This difference was especially great for students of low ability.

7. Students of high ability scored higher than students of average ability who in turn scored higher than those of low ability. Students of average ability experienced a sharp increase in scores on the sectioning tests between grades 5 and 6 whereas the same phenomenon occurred for those of low ability between grades 8 and 9.

8. The results of administering the sectioning tests in a group situation were similar to those obtained from

individual and small group administrations of previous studies. It is recalled that some modifications were made in the cuts and some solids were different.

9. The ability to section solids is not uni-factor. Some sectioning tasks may require a unique ability. Some evidence was presented suggesting that the shape of the section may be a factor in sectioning ability. For example, the ability required to identify sections which are ellipses may be different from those which are rectangles.

10. Neither the ability to draw plane figures nor the ability to recognize sections resulting from the oblique cut appeared to be a factor in sectioning ability.

11. A test on sectioning solids was useful for predicting geometry achievement in grade 5 and 8 geometry classes, particularly in grade 8 where the content was motion geometry. Some evidence that the tests were useful in grade 7 was also observed.

12. The utility of the sectioning tests in predicting geometry achievement in grades 9 and 10 was minimal.

Recommendations and Implications

In this study an attempt was made to build upon previous research on sectioning solids by using different solids, defining the cuts more precisely, and by using a group format of administering the test. The group method of testing would hopefully make it possible for the classroom teacher to make better use of the test, particularly in cases where the sectioning test predicted geometry achievement.

The results of the study, as they related to the questions asked, have been discussed and summarized in the previous sections of this chapter. To some extent the questions have been answered, yet other closely related questions have been raised. Opportunities for research related to this study include the following.

Other solids and cuts could be employed in future studies. Combinations of solids such as the sphere inside the cube (Chetverukhin, 1971) suggest that many variations are possible. The oblique cut on most solids can be constructed in several distinct ways. Do these different ways yield different results?

The ability to section solids appears to be very complex. More work is necessary to further analyze this ability. How does spatial ability as measured by the sectioning tests compare to sectioning ability from other spatial tests such as those dealing with embedded figures, coordination of viewpoints, or surface development?

How does the Van Hiele theory of the development of geometric thought apply to sectioning ability? Do stages in the ability to section solids coincide with the stages proposed by the Van Hieles? How do students process the information needed to solve the sectioning tasks? Is knowledge of deductive processes necessary for solving some tasks? These questions no doubt can best be investigated through individual testing and interview techniques.

Several studies on investigating the predictive power of the sectioning tests are suggested. Perhaps a sectioning

test composed of items different from those used in the present study could be constructed. Items with very high and very low difficulties could be eliminated. The sectioning tests might be more useful in predicting achievement for students of a particular ability level, or possibly could assist the teacher in identifying students who were very weak in spatial ability. Future studies might use different treatments of geometry at the grade 8 level in particular, or use a motion geometry treatment at other grade levels. It is indicated by the results of this study that the sectioning test was useful in predicting achievement in grade 8 motion geometry.

For the teacher it appears obvious that more experiences with solid figures need to be provided for children, particularly those of elementary school age. Many students did not know the names of even the most common solids. Experiences are needed where the students can estimate lengths, make freehand drawings of geometry figures, and observe equality and inequality in the length of sides and the congruence of angles. Angles, in particular, were very difficult for students to draw with any accuracy at all. Teachers should not assume that students can envision geometric sections prior to grade 9 and even then, the oblique sections have been mastered by only a few. Any study of the conics based on sections prior to the late secondary school years would be fruitless without first providing concrete experiences with the solids.

As for predicting achievement in geometry, the results

have not been conclusive. As suggested earlier the teacher of motion geometry in grade 8 might use the sectioning tests to identify students who would be likely to have problems. The teacher could then attempt to improve their sectioning ability with appropriate experiences (Bober, 1973) and determine whether or not these experiences aided the students in their study of motion geometry.

One final implication for the teachers is that giving the students an opportunity to do the sectioning tasks is a valuable activity in its own right. In every classroom tested in this study, the students appeared to enjoy the tasks. The tasks could easily give the teacher and students opportunity to discover and explore many of the fundamental properties of geometric figures.

BIBLIOGRAPHY

Bibliography

Adler, I. What shall we teach in high school geometry? The Mathematics Teacher, 1968, 61(3), 226-238.

Amick, D. J., and Crittenden, K. S. Analysis of variance and multivariate analysis of variance. In D. J. Amick and H. J. Walberg (Eds.), Introductory Multivariate Analysis. Berkeley: McCutchan Publishing Corporation, 1975.

Amick, D. J., and Walberg, H. J. Introductory Multivariate Analysis. Berkeley: McCutchan Publishing Corporation, 1975.

Armour, C. Geometry. Toronto: Gage, 1974.

Bay, K. Application of multivariate analysis of variance to educational and psychological research, part I: an introduction. Unpublished paper, Division of Educational Research Services, University of Alberta, Edmonton, 1969a.

Bay, K. Application of multivariate analysis of variance to educational and psychological research, part II: hypothesis testing by MANOVA programs. Unpublished paper, Division of Educational Research Services, University of Alberta, Edmonton, 1969b.

Bober, W. C. Role of maturity and experience in junior high school geometry. Unpublished master of education thesis, University of Alberta, 1973.

Boe, B. L. A study of the ability of secondary school pupils to perceive the plane sections of selected solid figures. Unpublished doctoral dissertation, University of Wisconsin, 1966.

Bottenberg, R. A., and Ward, J. H. Applied multiple linear regression. U.S. Department of Commerce Office of Technical Services, AD413128, 1963.

Bronfenbrenner, U. The experimental ecology of education. Educational Researcher, 1976, 5(9), 5-15.

Bryant, P. Perception and Understanding in Young Children. London: Methuen and Co. Ltd., 1974.

Carpenter, T. P., Coburn, T. G., Reys, R. E., Wilson, J. W. Results and implications of NAEP mathematics assessment: elementary school. The Arithmetic Teacher, 1975a, 22(6), 438-450.

Carpenter, T. P., Coburn, T. G., Reys, R. E., Wilson, J. W. Results and implications of NAEP mathematics assessment: secondary school. The Mathematics Teacher, 1975b, 68(6), 453-470.

Chetverukhin, N. F. An experimental investigation of pupils' spatial concepts and spatial imagination. In J. Kilpatrick and I. Wirszup (Eds.), Soviet Studies in the Psychology of Learning and Teaching Mathematics. Volume 5. Chicago: School Mathematics Study Group and the University of Chicago, 1971, 5-56.

Clifford, G. J. A history of the impact of research on teaching. In R. M. Travers (Ed.), Second Handbook of Research on Teaching. Chicago: Rand McNally, 1973.

Coleman, A. J., Edwards, G. D., and Beltzer, K. P. Mathematical Sciences in Canada. Science Council of Canada, Background Study No. 38. Ottawa: Science Council of Canada, 1975.

Copeland, R. W. How Children Learn Mathematics. New York: Macmillan Publishing Company, 1974.

Cronbach, L. J. Test validation. In R. L. Thorndike (Ed.), Educational Measurement. Washington: American Council on Education, 1971, 443-507.

D'Augustine, C. H. Factors relating to achievement with selected topics in geometry and topology. Arithmetic Teacher, 1966, 13(3), 192-197.

Davis, E. J. A study of the ability of school pupils to perceive the plane sections of selected solid figures. Unpublished doctoral dissertation, University of Florida, 1969.

Dean, J. E., Graham, W. R., and Moore, G. E. Principles of Mathematics 1. Toronto: Holt, Rinehart and Winston of Canada Ltd., 1970.

Dienes, Z. P., and Golding, E. W. Exploration of Space and Practical Measurement. London: The Educational Supply Association, 1966.

Dodwell, P. C. Children's understanding of spatial concepts. Canadian Journal of Psychology, 1963, 17, 141-161.

Dodwell, P. C. Children's perception and their understanding of geometrical ideas. In _Piagetian Cognitive-Development Research and Mathematical Education_. Washington, D.C.: National Council of Teachers of Mathematics, 1971, 178-188.

Douglass, H. R. The prediction of pupil success in high school mathematics. _Mathematics Teacher_, 1935, _28_, 489-504.

Douglass, H. R., and Kinney, L. B. Mathematics. _Review of Educational Research_, 1938, _8_, 51-57.

Draper, N. R., and Smith, H. _Applied Regression Analysis_. New York: John Wiley and Sons, 1966.

Duckworth, E. _Piaget rediscovered_. A report on the conference of March, 1964, Cognitive studies and curriculum development. Cornell University and University of California, 1964.

Eicholz, R. E., and O'Daffer, P. G. _Elementary School Mathematics Book 5_. (2nd ed.) Don Mills, Ontario: Addison Wesley, 1969a.

Eicholz, R. E., and O'Daffer, P. G. _Elementary School Mathematics Book 6_. (2nd ed.) Don Mills, Ontario: Addison Wesley, 1969b.

Eliot, J., and Salkind, N. J. _Children's Spatial Development_. Springfield: Charles C. Thomas, 1975.

Fawcett, H. P. The geometric continuum. _The Mathematics Teacher_, 1970, _63_(5), 411-419.

Fehr, H. F. The present year-long course in Euclidean geometry must go. _The Mathematics Teacher_, 1972, _65_(2), 102, 151-154.

Fennema, E. Sex differences in mathematics learning: why??? _Elementary School Journal_, 1974, 75(12), 183-190.

Ferguson, G. A. _Statistical Analysis in Psychology and Education_. New York: McGraw-Hill, 1976.

Flavell, J. H. _The Developmental Psychology of Jean Piaget_. Princeton, New Jersey: D. Van Nostrand, 1963.

Gearhart, G. What do mathematics teachers think about the high school geometry controversy. The Mathematics Teacher, 1975, 68(6), 486-493.

Geometry kindergarten to grade thirteen, report of the (K-13) geometry committee. Toronto: The Ontario Institute for Studies in Education, 1967.

Goals for school mathematics. The Report of the Cambridge Conference on School Mathematics. Published for Educational Services Incorporated by Houghton Mifflin Co., Boston. 1963.

Guay, R. B, and McDaniel, E. D. The relationship between mathematics achievement and spatial abilities among elementary school children. Journal for Research in Mathematics Education, 1977, 8(3), 211-215.

Gulliksen, H. Theory of Mental Tests. New York: Wiley and Sons, 1965.

Hanna, G. S. A summary of the literature of geometry prediction with emphasis upon methodology and theory. School Science and Mathematics, 1966, 66, 723-728.

Harman, H. H. Modern Factor Analysis. Chicago: The University of Chicago Press, 1967.

Harrison, D. B. Reflective intelligence and mathematics learning. Unpublished doctoral dissertation, University of Alberta, 1967.

Henderson, K. B. (Ed.). Geometry in the Mathematics Curriculum. 36th Yearbook on the National Council of Teachers of Mathematics, Reston: N.C.T.M., 1973.

Holloway, G. E. T. An Introduction to "The Child's Conception of Space." London: Routledge and Kegan Paul, 1967.

Hummell, J. J., and Sligo, J. R. Empirical comparison of univariate and multivariate analysis of variance procedures. Psychological Bulletin, 1971, 76(1), 49-57.

Junior high mathematics consortium - geometry unit grade 7. Edmonton School District No. 7, 1975.

Junior high mathematics consortium - geometry unit grade 8. Edmonton School District No. 7, 1975.

Junior high mathematics consortium - geometry unit grade 9. Edmonton School District No. 7, 1975.

Kapadia, R. A critical examination of Piaget-Inhelder's view on topology. Educational Studies in Mathematics, 1974, 5, 419-424.

Kerlinger, F. N., and Pedhazur, E. J. Multiple Regression in Behavioral Research. New York: Holt, Rinehart, and Winston, 1973.

Kidder, F. R. Elementary and middle school children's comprehension of Euclidean transformations. Journal for Research in Mathematics Education, 1976, 7(1), 40-52.

Kidder, F. R. Euclidean transformations: elementary school spaceometry. The Arithmetic Teacher, 1977, 24(3), 201-207.

Korth, B. Exploratory factor analysis. In D. J. Amick and H. J. Walberg (Eds.), Introductory Multivariate Analysis. Berkeley: McCutchan Publishing Corporation, 1975.

Kuper, M. The effects of model building on space perception. Unpublished paper, University of Alberta, 1975.

Kuper, M., and Walter, M. Making rectangular solids. Jerusalem, Israel: The Ministry of Education and Culture, 1975.

Kuper, M., and Walter, M. From edges to solids. Mathematics Teaching, 1976, No. 74, 20-23.

Laurendeau, M., and Pinard, A. The Development of the Concept of Space in the Child. New York: International Universities Inc., 1970.

Little, J. J. A mathematical and cognitive analysis of children's behavior in spatial problems. Unpublished doctoral dissertation, University of Alberta, 1976.

Lovell, K. R. A follow-up study of some aspects of the work of Piaget and Inhelder on the child's conception of space. British Journal of Educational Psychology, 1959, 29, 104-117.

Lovell, K. R. The Growth of Basic Mathematical and Scientific Concepts in Children. (4th ed.) New York: Philosophical Library, 1965.

Lovell, K. R. The Growth of Understanding in Mathematics: Kindergarten through Grade Three. New York: Holt, Rinehart and Winston, 1971.

Lovell, K. R., Healy, D., and Rowland, A. D. Growth of some geometrical concepts. Child Development, 1962, 33, 751-767.

Lesh, R. Transformation geometry in elementary school. In J. L. Martin (Ed.), Space and Geometry. Columbus, Ohio: ERIC/SMEAC Science, Mathematics, and Environmental Education Information Analysis, 1976.

Martin, J. L. An analysis of some of Piaget's topological tasks from a mathematical point of view. Journal for Research in Mathematics Education, 1976a, 7(1), 8-24.

Martin, J. L. A test with selected topological properties of Piaget's hypothesis concerning the spatial representation of the young child. Journal for Research in Mathematics Education, 1976b, 7(1), 26-38.

Martin, J. L. The Erlanger Program as a model for the child's conception of space. In A. R. Osborne (Ed.), Models for Learning Mathematics. Columbus, Ohio: ERIC/SMEAC Science, Mathematics, and Environmental Education Information Analysis, 1976c.

Martin, J. L. (Ed.). Space and Geometry. Columbus, Ohio: ERIC/SMEAC Science, Mathematics, and Environmental Education Information Analysis, 1976d.

Mathematics in primary schools. London: Her Majesty's Stationary Office, 1965.

Mitchelmore, M. C. Cross-cultural research on concepts of space and geometry. In J. L. Martin (Ed.), Space and Geometry. Columbus, Ohio: ERIC/SMEAC Science, Mathematics, and Environmental Education Information Analysis, 1976.

Montangero, J. Recent research on the child's conception of space and geometry in Geneva: research work on spatial concepts at the International Center for Genetic Epistemology. In J. L. Martin (Ed.), Space and Geometry. Columbus, Ohio: ERIC/SMEAC Science, Mathematics, and Environmental Education Information Analysis, 1976.

Morrison, D. F. Multivariate Statistical Methods. Toronto: McGraw-Hill, 1967.

Mulaik, S. A. The Foundations of Factor Analysis. New York: McGraw-Hill, 1972.

Olson, D. R. Cognitive Development The Child's Acquisition of Diagonality. New York: Academic Press, 1970.

Ong, S. T. A curriculum implementation of creative problem solving in junior high school mathematics. Unpublished doctoral dissertation, University of Alberta, 1976.

Overview and analysis of school mathematics. Washington: Conference Board of the Mathematical Sciences, 1975.

Page, E. I. Haptic perception: a consideration of one of the investigations of Piaget and Inhelder. Education Review, 1959, 11, 115-124.

Palow, W. P. A study of the ability of public school students to visualize particular perspectives of selected solid figures. Science and Mathematics Education Information Report, Research Reporting Sections, N.C.T.M., 1970.

Piaget, J. Cognitive development in children: the Piaget papers. In R. E. Ripple and V. N. Rockcastle (Eds.), Piaget Rediscovered. Ithica: Cornell University Press, 1964.

Piaget, J. Comments on mathematical education. In A. G. Howson (Ed.), Developments in Mathematical Education. Porceedings of the Second International Congress on Mathematical Education. London: Cambridge University Press, 1973.

Piaget, J. and Inhelder, B. The Child's Conception of Space. Translated by F. J. Langdon and J. L. Lunzer. New York: Norton, 1967. Original French edition, 1948: First published in English, 1956.

Piaget, J., Inhelder, B. and Smeninska, A. The Child's Conception of Geometry. Translated by E. A. Lunzer. New York: Basic Books, 1960. Original French edition, 1948: First published in English, 1960.

Pothier, Y. M. Classroom testing of geometric ability. Unpublished master's colloquium paper, University of Alberta, 1975.

Riggs, F. Some aspects of spatial abilities and elementary school mathematics. Unpublished doctoral dissertation. University of Alberta, 1972.

Rivoire, J. L. Development of reference systems in children. Unpublished doctoral dissertation, Department of Philosophy and Psychology, University of Arizona, 1961.

Robinson, E. Mathematical foundations of the development of spatial and geometrical concepts. In J. L. Martin (Ed.), Space and Geometry. Columbus, Ohio: ERIC/SMEAC Science, Mathematics, and Environmental Education Information Analysis, 1976.

Sauvy, J. and Sauvy, S. The Child's Discovery of Space. Baltimore: Penquin, 1974.

Smith, I. M. Spatial Ability Its Educational and Social Significance. London: University of London Press, 1964.

Smock, C. D. Piaget's thinking about the development of space concepts and geometry. In J. L. Martin (Ed.), Space and Geometry. Columbus, Ohio: ERIC/SMEAC Science, Mathematics, and Environmental Education Analysis, 1976.

Sowder, L. High versus low geometry achievers in the NLSMA Y population, Journal for Research in Mathematics Education, 1974, 5(1), 20-27.

Tatsuoka, M. M. Validation Studies. The Use of Multiple Regression Equations. Champaign, Illinois: Institute for Personality and Ability Testing, 1969.

Tatsuoka, M. M. Significance Tests. Univariate and Multivariate. Champaign, Illinois: Institute for Personality and Ability Testing, 1971.

Timm, N. H. Multivariate Analysis with Applications in Education and Psychology. Belmont, California: Wadsworth Publishing Co., 1975.

Trafton, P. R., and LeBlanc, J. F. Informal geometry in grades K-6. In K. B. Henderson (Ed.), Geometry in the Mathematics Curriculum. 36th Yearbook of the National Council of Teachers of Mathematics. Reston, Virginia: National Council of Teachers of Mathematics, 1973.

Williford, H. What does research say about geometry in the elementary school? The Arithmetic Teacher, 1972, 19, 97-104.

Wilcox, M. S. Geometry A Modern Approach. Toronto: Addison Wesley, 1968.

Winer, N. J. Statistical Principles in Experimental Design. New York: McGraw-Hill, 1962.

Wirszup, I. Breakthroughs in the psychology of learning and teaching geometry. In J. L. Martin (Ed.), Space and Geometry. Columbus, Ohio: ERIC/SMEAC Science, Mathematics, and Environmental Education Analysis, 1976.

APPENDICES

APPENDIX 1

The Sectioning Tests

Protocol for Administering Test I

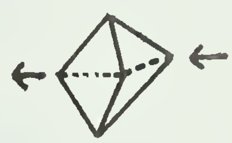
1. Distribute the answer booklets and have the students complete the first page. In the space for grade include the teacher's name as well as the grade. Some explanation may be necessary concerning the age with regard to months.

2. Show the students the complete sphere and discuss its properties briefly. Ask the students what plane shape will be seen if you cut the sphere into two parts. Illustrate with the cut sphere and draw a circle on the blackboard freehand. Have the students do the same on their paper below the information they have completed.

3. The students now turn to page 2 of their booklet.

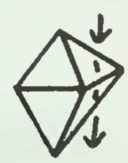
4. Illustrate the complete octahedron and discuss its properties briefly. How many faces, how many edges, etc.

5. Perform the parallel cut on the octahedron using a ruler in place of a knife. Ask the students what plane shape would result if you were able to look at the cut pieces. Use the cut solid to indicate the resulting plane figure is a square. Have them draw <u>freehand</u> a square in the box labelled A on page 2. You draw one on the board.

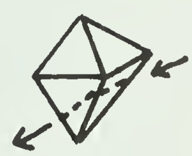


6. Repeat step 5 with a longitudinal cut. The result is a rhombus. Have the students draw a rhombus in box B.

7. Repeat step 5 with a transverse cut. The result is a hexagon. Have the students draw a hexagon in box C.



8. Repeat step 5 with an oblique cut. The result is a trapezoid. Have the students draw a trapezoid in box D.



9. Indicate to the students that in the first part of the exercise, you will illustrate similar cuts as executed above on the following solids: cube, triangular prism, cone and parallelepiped. Show the solids briefly. Also, without the solids, indicate the general direction of the four cuts as being:
a) perpendicular to the floor and pointing at the student,
b) perpendicular to the floor and facing the student,
c) parallel to the floor,
d) oblique to the floor.

10. Points of emphasis

a) The students are to draw the plane surface they would see if the solid were taken apart. In the exercise the cut will be illustrated but the solid not taken apart.

b) The students are to make the drawings freehand being careful to make lines approximately equal when that is the case, to make corners square when that is the case, and lines straight if they are straight lines, and parallel if they are parallel lines.

c) If the students wish the cut illustrated again, do it immediately but do not return to a cut later.

d) Make sure the cut is being drawn in the proper place on the answer sheet.

e) The tester should make the cut and turn to face the left and right of the class so every student sees the cuts face on.

11. QUESTIONS??

12. Illustrate the 16 cuts as indicated on the sheet labelled - TEST ID.

13. Distribute the multiple choice distractors. The students are not to mark on their paper. Explain that each cut will be illustrated once again and the student is to select the answer he thinks is correct and circle the appropriate letter on the answer sheet. If you think the answer is not on the sheet, circle F. Take care that the students are considering the correct set of distractors after each cut.

14. QUESTIONS??

15. Illustrate the 16 cuts as indicated on the sheet labelled - TEST IMC.

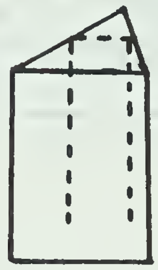
16. Collect the answer sheets.

TEST ID

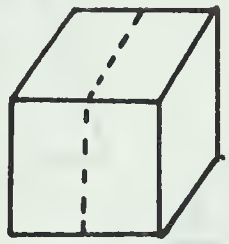
The cuts are presented in the following order:

1. parallelepiped: longitudinal

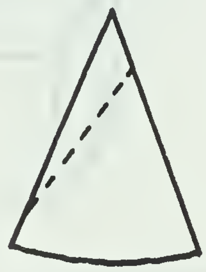
2. triangular prism: transverse



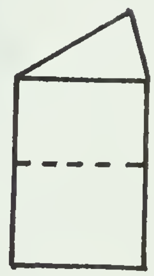
3. cube: longitudinal



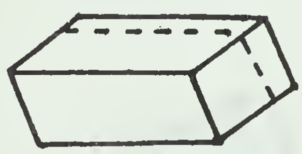
4. cone: oblique



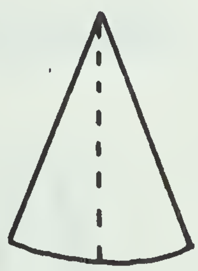
5. triangular prism: parallel



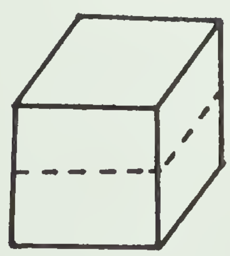
6. parallelepiped: transverse



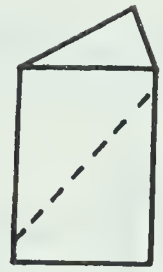
7. cone: longitudinal



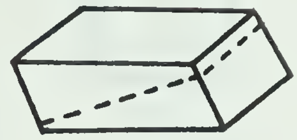
8. cube: parallel



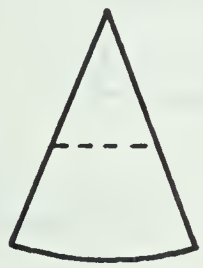
9. triangular prism: oblique



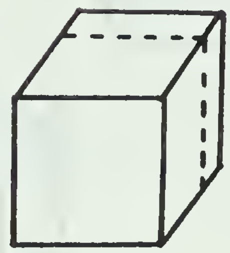
10. parallelepiped: oblique



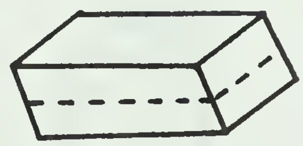
11. cone: parallel



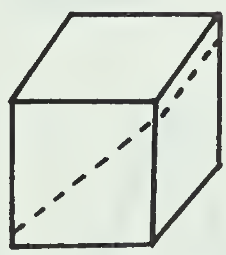
12. cube: transverse



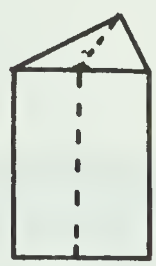
13. parallelepiped: parallel



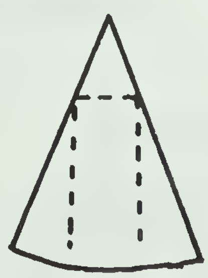
14. cube: oblique



15. triangular prism: longitudinal



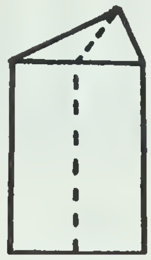
16. cone: transverse



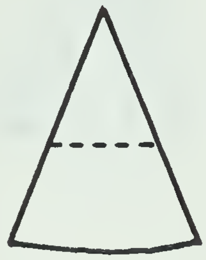
TEST IMC

The cuts are presented in the following order:

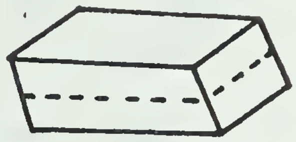
1. triangular prism: longitudinal



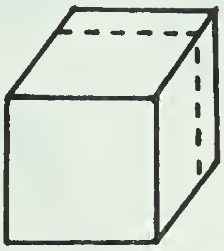
2. cone: parallel



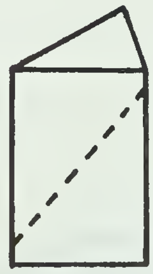
3. parallelepiped: parallel



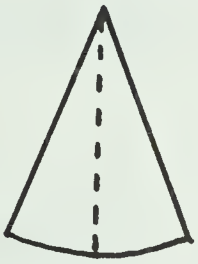
4. cube: transverse



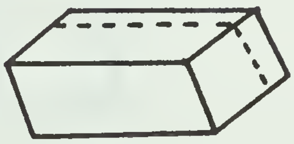
5. triangular prism: oblique



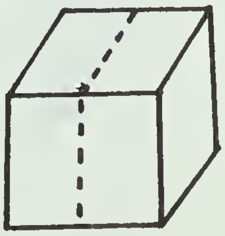
6. cone: longitudinal



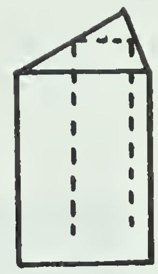
7. parallelepiped: transverse



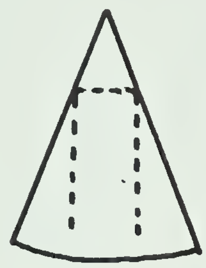
8. cube: longitudinal



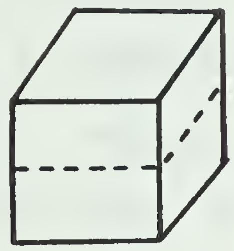
9. triangular prism: transverse



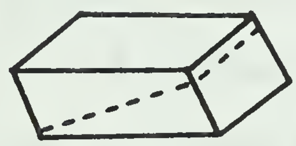
10. cone: transverse



11. cube: parallel



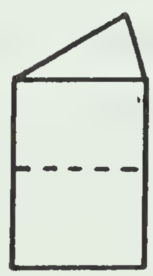
12. parallelepiped: oblique



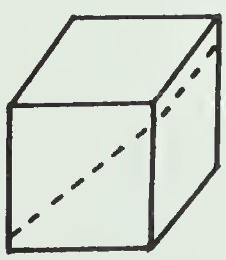
13. cone: oblique

14. parallelepiped: longitudinal

15. triangular prism: parallel



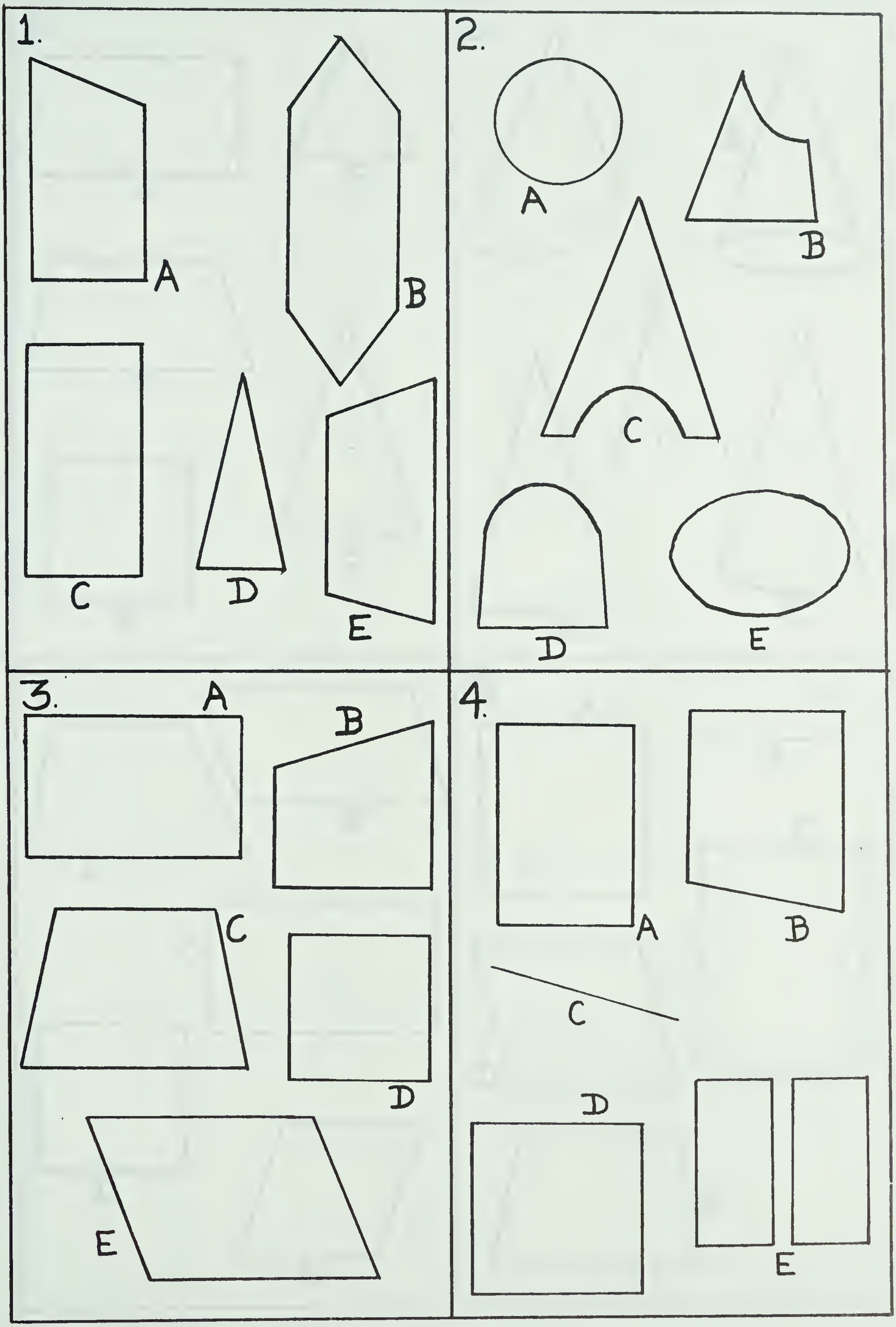
16. cube: oblique



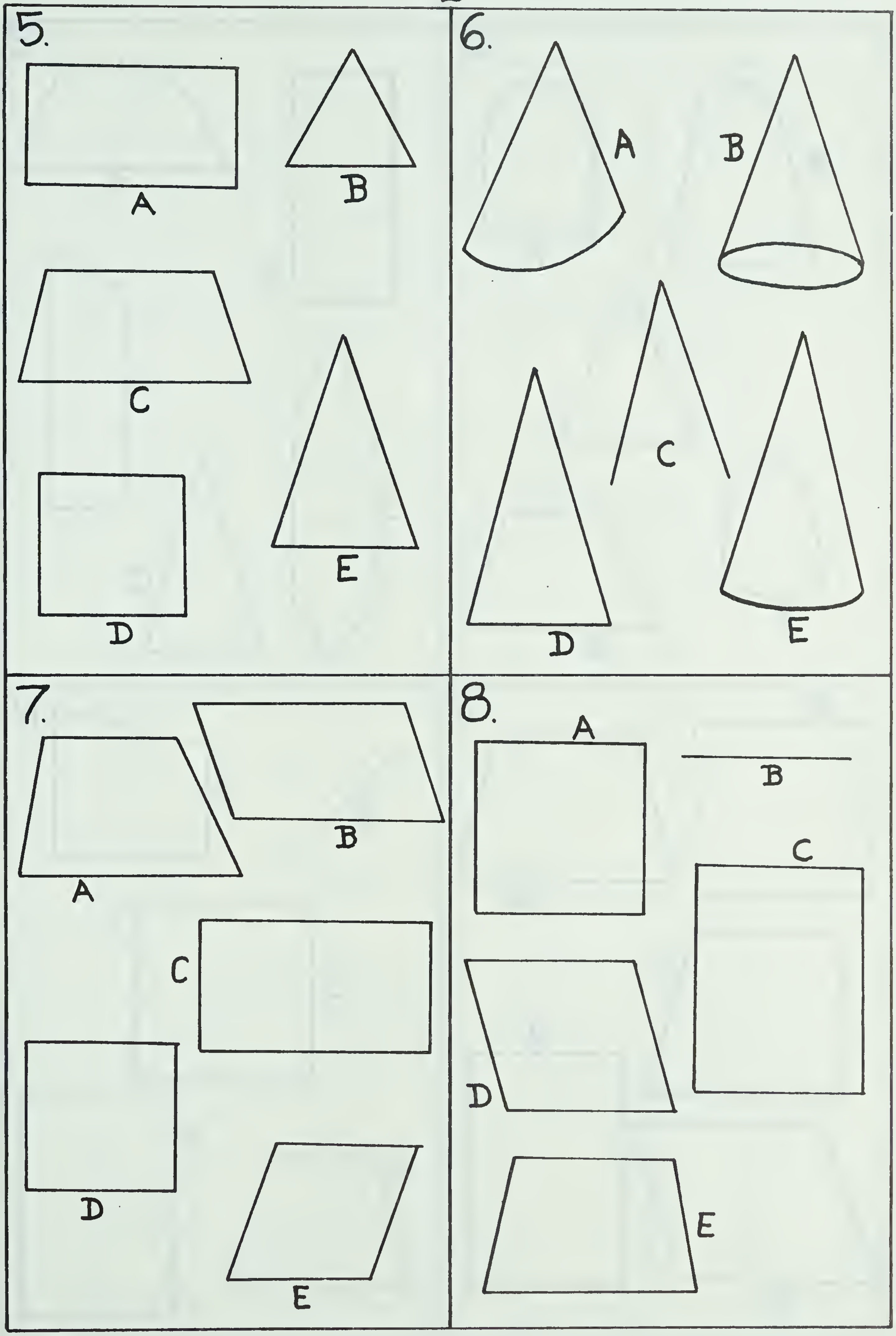
CORRECT ANSWERS

Test ID		Test IMC		
1.	parallelogram	1.	C	rectangle
2.	rectangle	2.	A	circle
3.	square	3.	E	parallelogram
4.	ellipse	4.	D	square
5.	equilateral triangle	5.	F	none of these
6.	parallelogram	6.	D	isosceles triangle
7.	isosceles triangle	7.	B	parallelogram
8.	square	8.	A	square
9.	isosceles triangle	9.	B	rectangle
10.	parallelogram	10.	E	one-half hyperbola
11.	circle	11.	C	square
12.	square	12.	E	parallelogram
13.	parallelogram	13.	D	ellipse
14.	rectangle	14.	C	parallelogram
15.	rectangle	15.	E	equilateral triangle
16.	one-half hyperbola	16.	D	rectangle

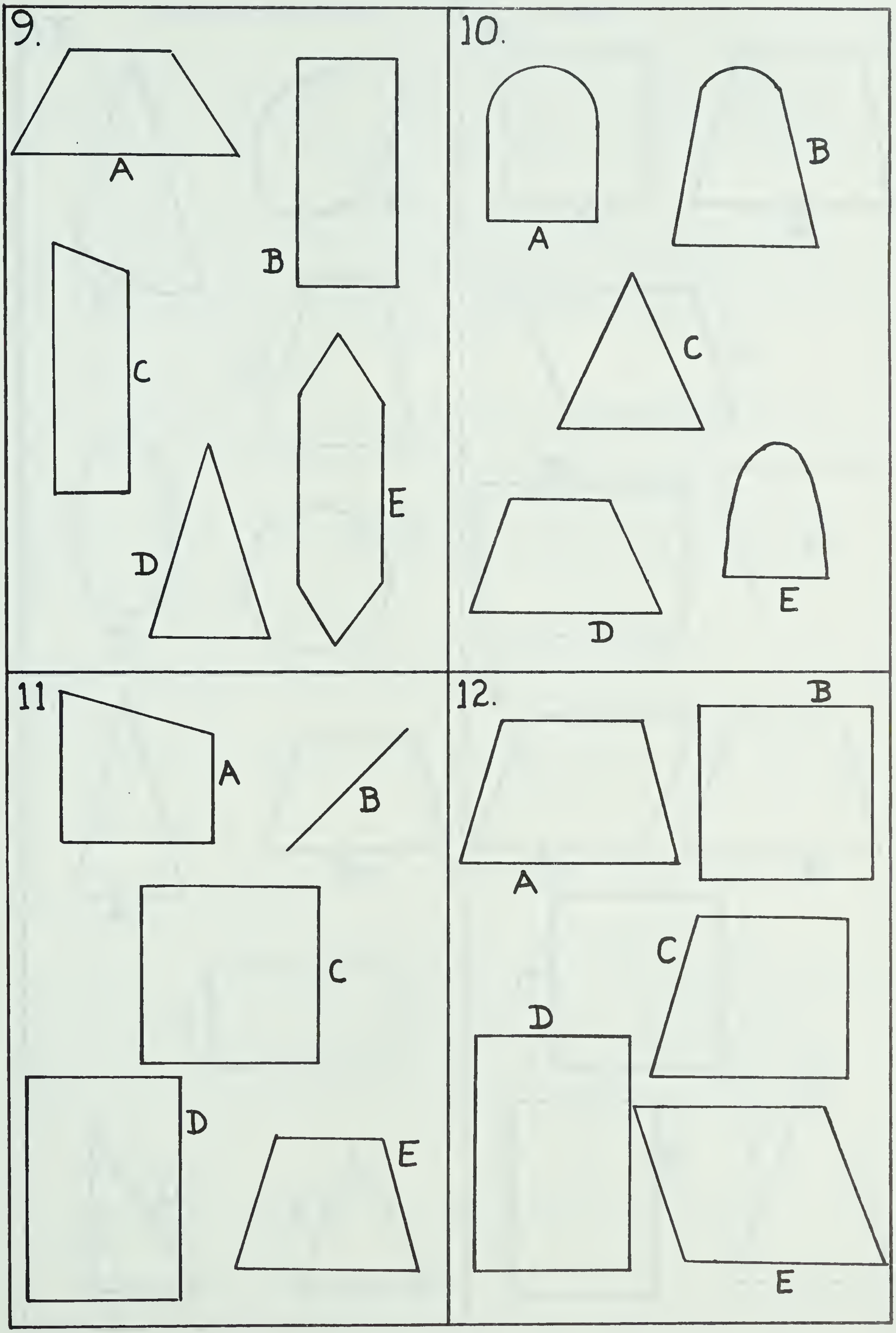
1



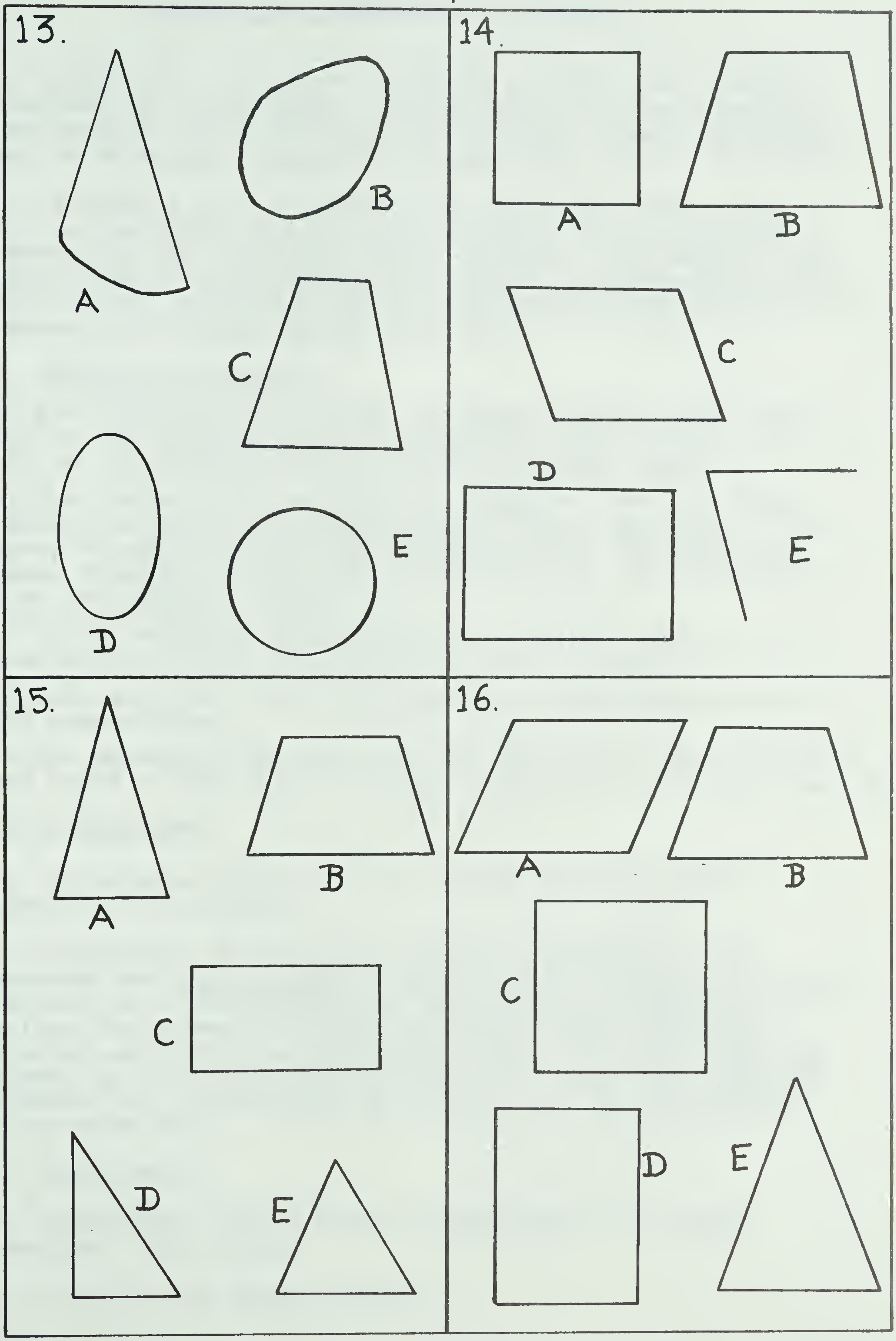
2



3



4



Protocol for Administering Test II

1. Distribute the answer booklets and have the students complete the first page. In the space for grade include the teacher's name as well as the grade. Some explanation may be necessary concerning the age with regard to months.

2. Briefly review the nature of the test. Illustrate a cut on the sphere and the longitudinal cut on the octahedron and the resulting plane surfaces. If students have difficulty in recollecting the nature of the test, further cuts on the octahedron may be illustrated (see steps 2-9 on protocol for administering Test I).

3. Points of emphasis

a) The students are to draw the plane surface they would see if the solid were taken apart. In the exercise the cut will be illustrated but the solid not taken apart.

b) The students are to make the drawings freehand being careful to make lines approximately equal when that is the case, to make corners square when that is the case, and lines straight if they are straight lines, and parallel if they are parallel lines.

c) If the students wish the cut illustrated again, do it immediately but do not return to a cut later.

d) Make sure the cut is being drawn in the proper place on the answer sheet.

e) The tester should make the cut and turn to face the left and right of the class so every student sees the cuts face on.

4. QUESTIONS??

5. Illustrate the 16 cuts as indicated on the sheet labelled - Test IID.

6. Distribute the multiple choice distractors. The students are not to mark on their paper. Explain that each cut will be illustrated once again and the student is to select the answer he thinks is correct and circle the appropriate letter on the answer sheet. If you think the answer is not on the sheet, circle F. Take care that the students are considering the correct set of distractors after each cut.

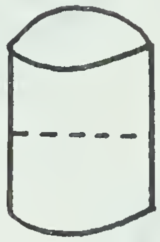
7. QUESTIONS??

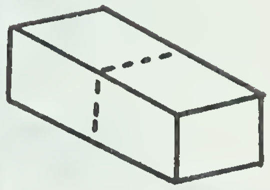
8. Illustrate the 16 cuts as indicated on the sheet labelled - Test IIMC.

9. Collect the answer sheets.

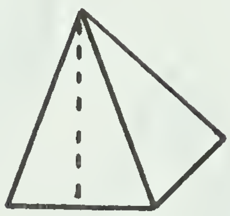
Test IID

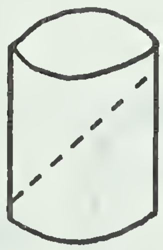
The cuts are presented in the following order:

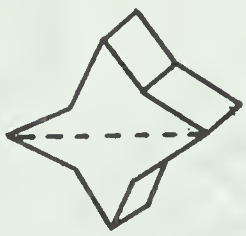
1. cylinder: parallel 

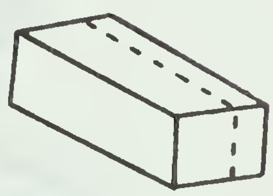
2. rectangular prism: longitudinal 

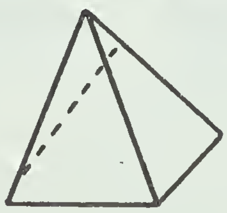
3. star: transverse 

4. pyramid: longitudinal 

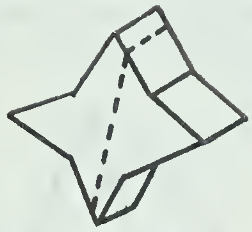
5. cylinder: oblique 

6. star: parallel 

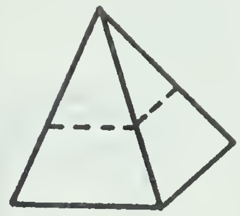
7. rectangular prism: transverse 

8. pyramid: oblique 

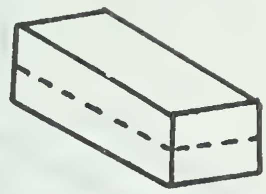
9. star: oblique



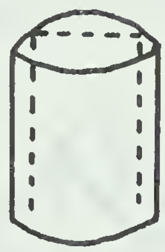
10. pyramid: parallel



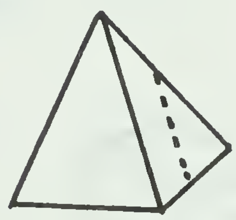
11. rectangular prism: parallel



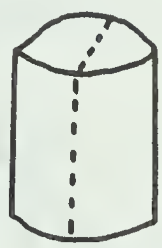
12. cylinder: transverse



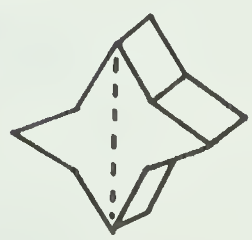
13. pyramid: transverse



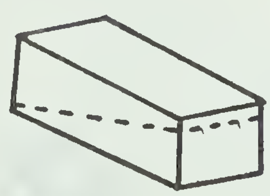
14. cylinder: longitudinal



15. star: longitudinal



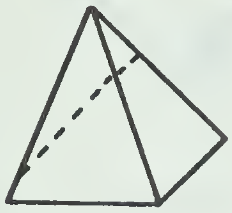
16. rectangular prism: oblique



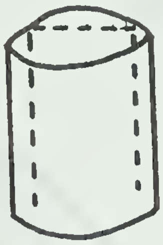
Test IIMC

The cuts are presented in the following order:

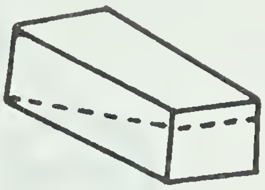
1. pyramid: oblique



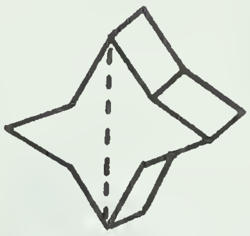
2. cylinder: transverse



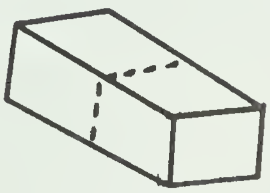
3. rectangular prism: oblique



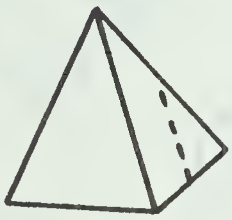
4. star: longitudinal



5. rectangular prism: longitudinal

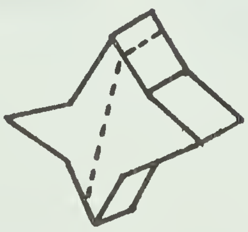


6. pyramid: transverse

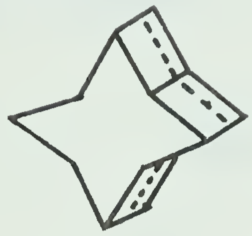


7. cylinder: oblique

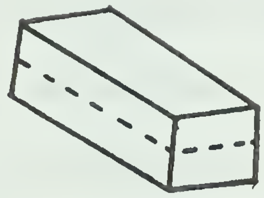
8. star: oblique



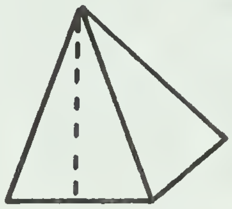
9. star: transverse



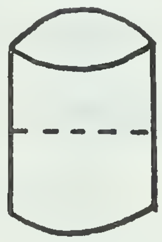
10. rectangular prism: parallel



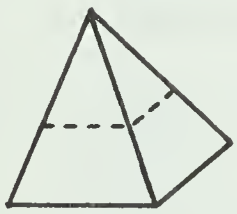
11. pyramid: longitudinal



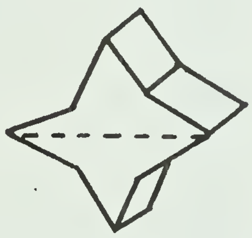
12. cylinder: parallel



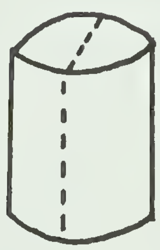
13. pyramid: parallel



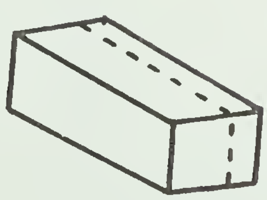
14. star: parallel



15. cylinder: longitudinal



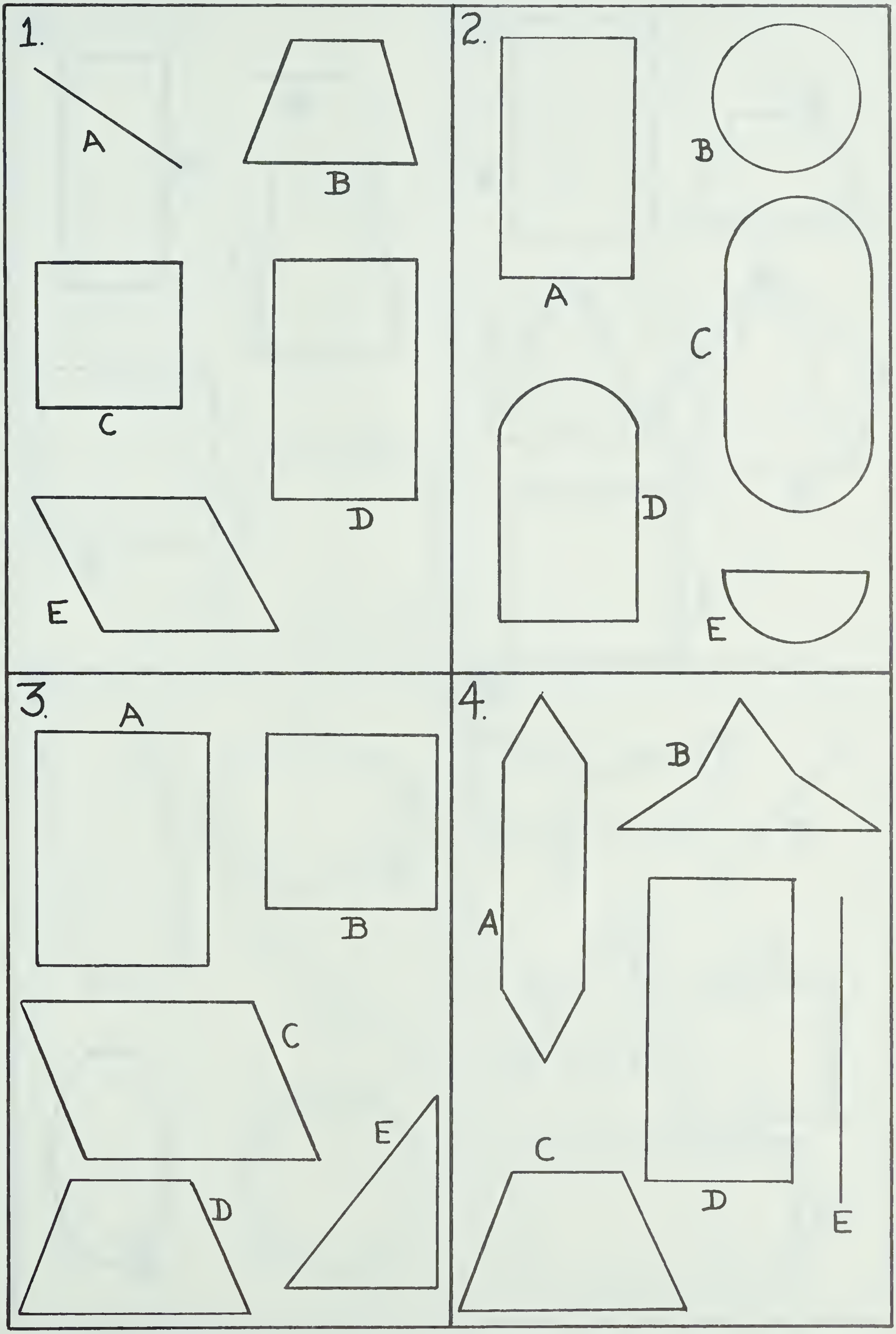
16. rectangular prism: transverse



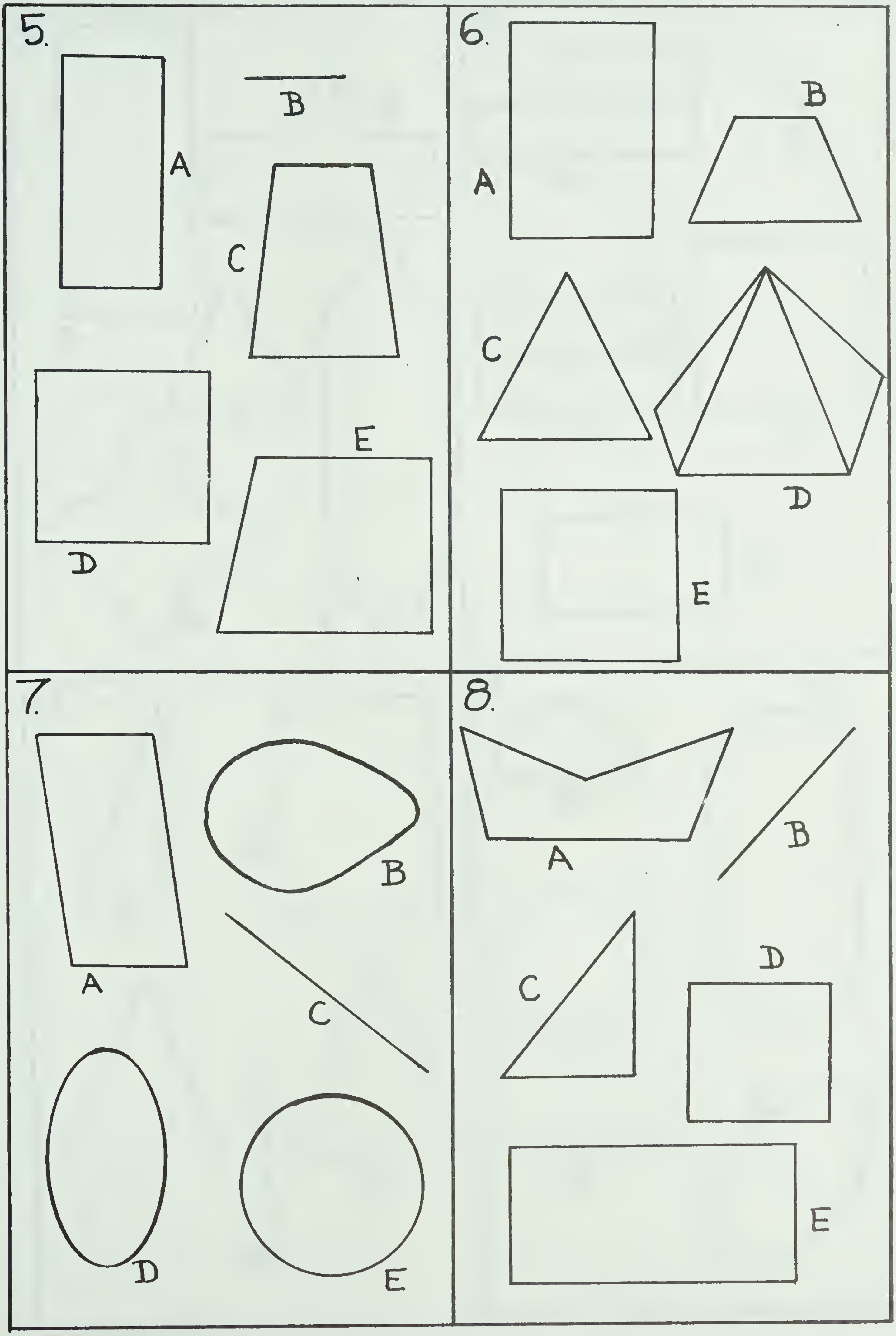
CORRECT ANSWERS

Test IID		Test IIMC		
1.	circle	1.	B	isosceles trapezoid
2.	rectangle	2.	A	rectangle
3.	star	3.	A	rectangle
4.	isosceles triangle	4.	D	rectangle
5.	ellipse	5.	A	rectangle
6.	rectangle	6.	B	isosceles trapezoid
7.	rectangle	7.	D	ellipse
8.	isosceles trapezoid	8.	E	rectangle
9.	rectangle	9.	D	star
10.	square	10.	E	rectangle
11.	rectangle	11.	E	isosceles triangle
12.	rectangle	12.	C	circle
13.	isosceles trapezoid	13.	E	square
14.	rectangle	14.	A	rectangle
15.	rectangle	15.	E	rectangle
16.	rectangle	16.	C	rectangle

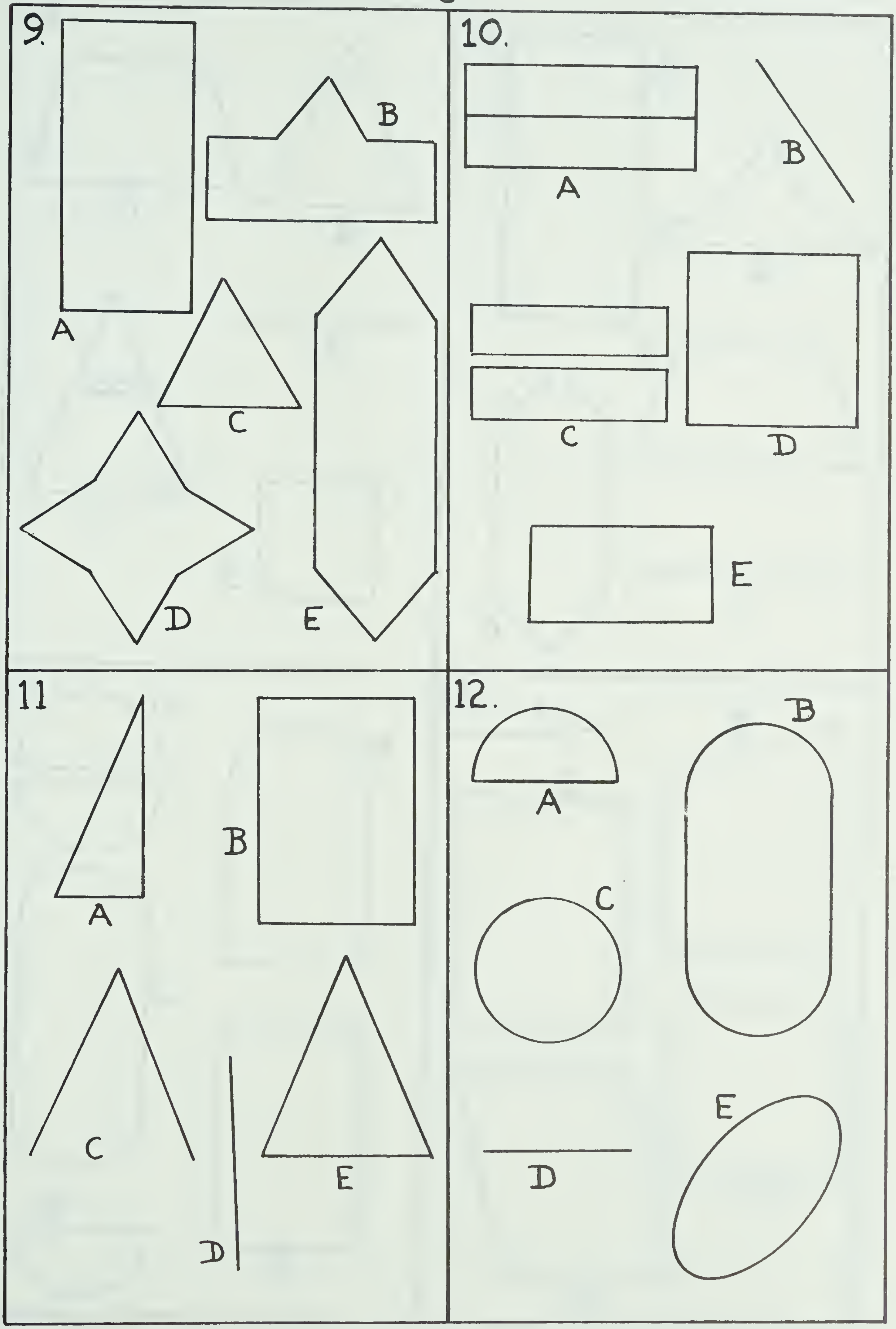
1



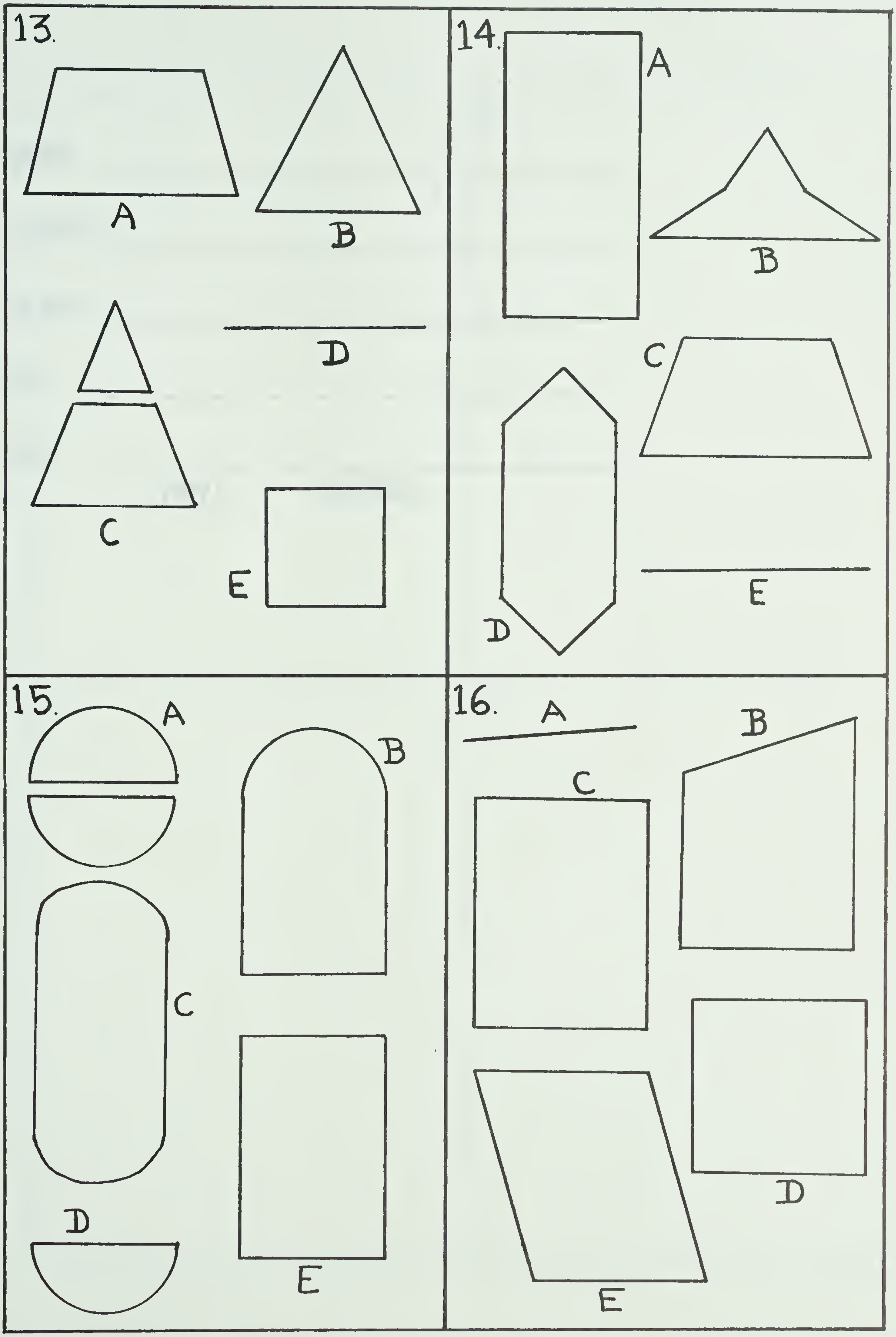
2



3



4



NAME ______________________________

SCHOOL ______________________________

GRADE ______________________________

SEX ______________________________

AGE ______________________________
years months

A

B

C

D

1.	2.
3.	4.

5.

6.

7.

8.

9.

10.

11

12.

13.

14.

15.

16.

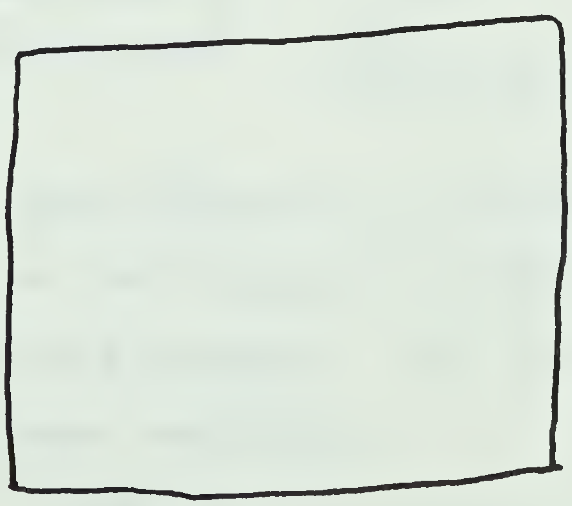
1.	A	B	C	D	E	F
2.	A	B	C	D	E	F
3.	A	B	C	D	E	F
4.	A	B	C	D	E	F
5.	A	B	C	D	E	F
6.	A	B	C	D	E	F
7.	A	B	C	D	E	F
8.	A	B	C	D	E	F
9.	A	B	C	D	E	F
10.	A	B	C	D	E	F
11.	A	B	C	D	E	F
12.	A	B	C	D	E	F
13.	A	B	C	D	E	F
14.	A	B	C	D	E	F
15.	A	B	C	D	E	F
16.	A	B	C	D	E	F

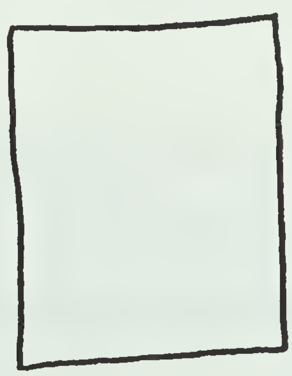
APPENDIX 2

Illustrated Examples of Common Errors on Sectioning Tests

In the majority of cases drawings were not difficult to score as correct or incorrect. Most correct drawings were very good representations of the section, their accuracy falling well within the criteria presented in Chapter III. Incorrect responses often included the drawing of a figure quite different from the correct response. The sets of multiple choice distractors presented in Appendix I were constructed from common incorrect responses given on previous sectioning tests. The examples presented on the following pages represent instances of drawings which were scored as incorrect. The drawings did not meet the criteria for correct representations yet the errors were such that in many instances it might have been assumed that the child could form a correct mental image of the section and not represent it correctly as a drawing.

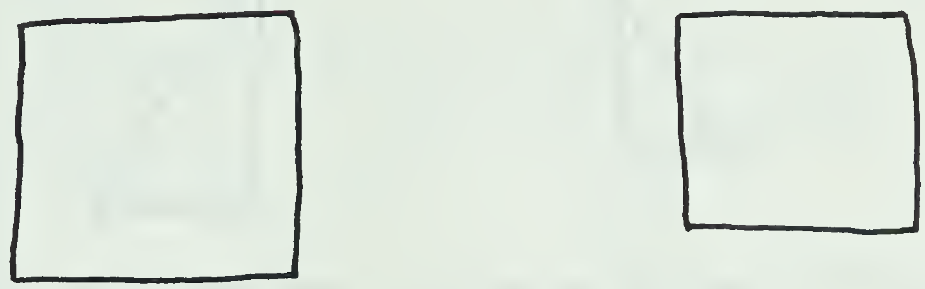
The most common error for drawing squares was to construct the adjacent sides of different lengths. The two examples below illustrate this error.





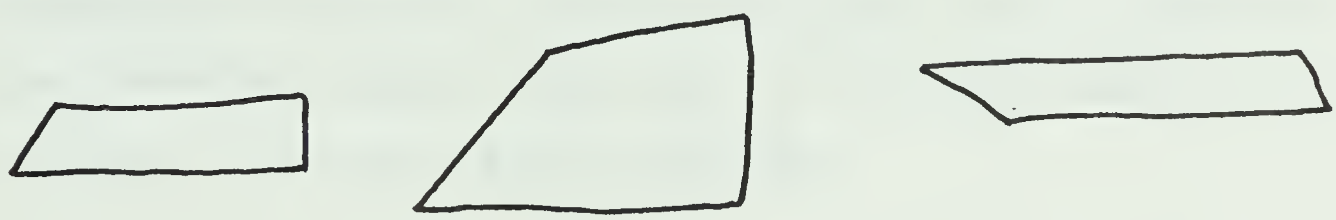
The sections which were non-square rectangles were usually very easy to recognize. The exception to this was

the rectangle resulting from the oblique cut on the cube. Here the representation often appeared to be a square. The following drawings were judged not to be non-square rectangles.

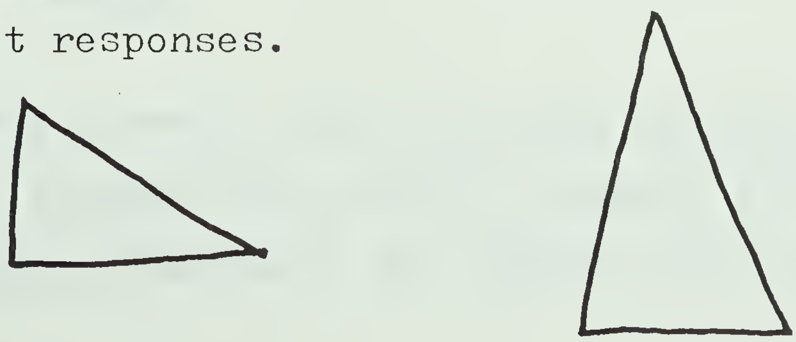


The rectangles resulting from sections on the cylinder often had curvatures in the drawings.

The most common error in drawing parallelograms was to illustrate opposite sides with lines of unequal slope. In many cases one of the base angles was a right angle while the other was obtuse or acute.

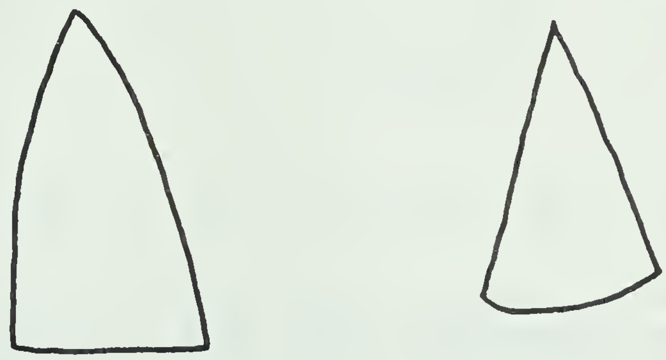


The equilateral triangle resulting from the parallel cut on the triangular prism was often represented by an isosceles triangle. The following drawings are examples of incorrect responses.

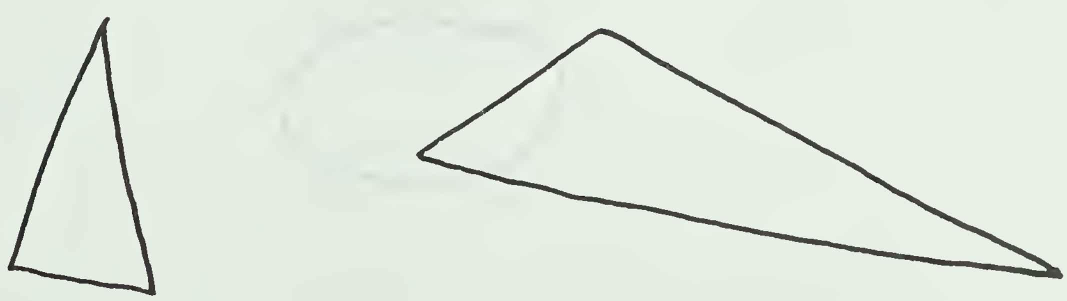


The isosceles triangles resulting from the longitudinal cuts on the cone and the pyramid were often constructed with a right angle as one of the base angles.

On the cone, the isosceles triangle often included curved sides or a curved base.

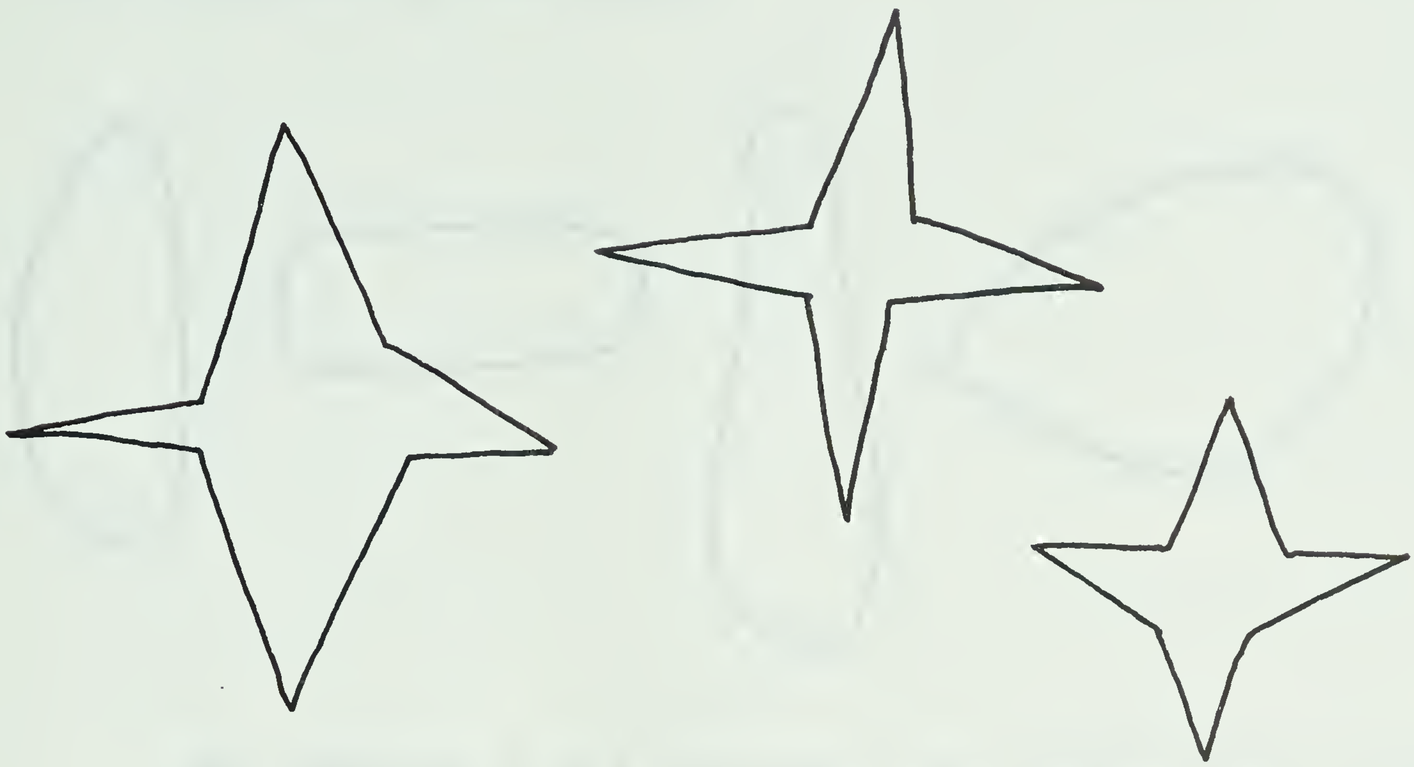


The isosceles triangle resulting from the oblique cut on the triangular prism was one of the most difficult to draw. The vertical angle was often drawn less than 60°, an impossible situation. The second error was to construct the "vertical angle" very near to a straight angle, with the triangle not really being isosceles.



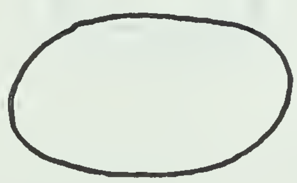
Many students had difficulty drawing the star. Many figures included four points, however all or some of the triangular portions of the figure were isosceles. Very

little attention was paid to angle properties in many instances.



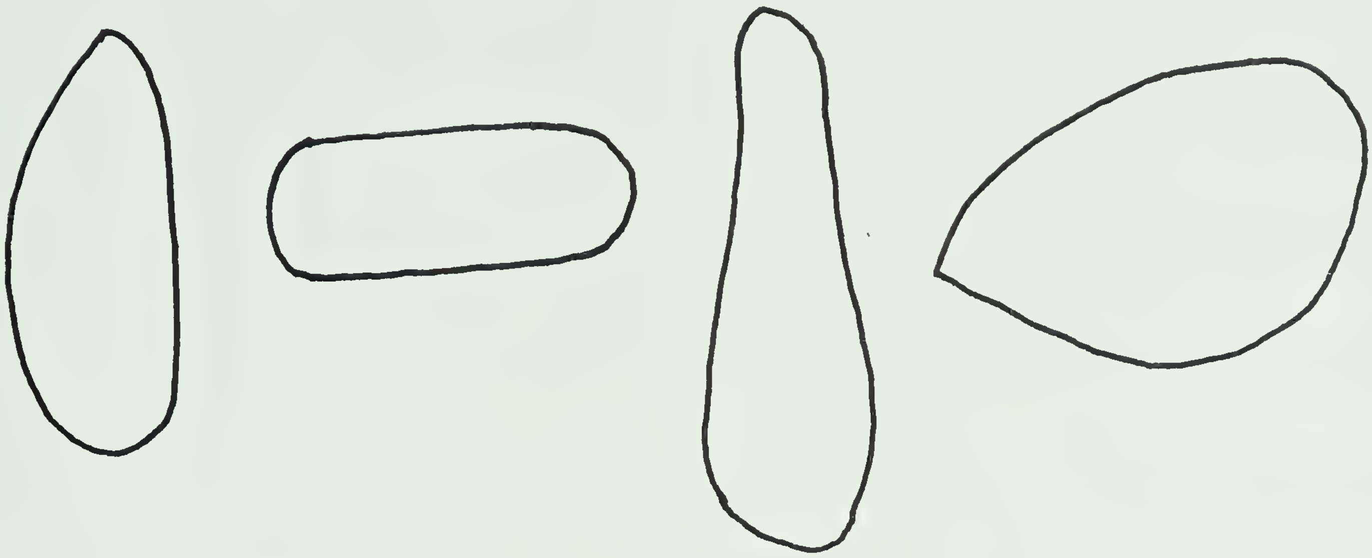
The sections which were trapezoids were almost always either drawn correctly or drawn as different figures such as a parallelogram or triangle.

Circles were also usually drawn well within the accepted criteria. The errors noted were similar to the following figure, which was judged to be more like an ellipse.



Ellipses were often drawn as egg shaped for the oblique cut on the cone or as rectangular shapes with curved

extremities. Most difficulties arose in attempting to draw the curvature of the figure.



The drawings of the hyperbola also included many errors in the curvature of the sides or the vertex of the figure. Straight sides and a peaked vertex were common features of these drawings. In many instances the sides were convex rather than concave.

APPENDIX 3

Item Statistics on Test I

and Test II

Table 39

Correlations Among Items in Test ID

		S1				S2				S3				S4			
		C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
S1	C1	1															
	C2	.18	1														
	C3	.33	.25	1													
	C4	.10	.08	.14	1												
S2	C1	.19	.17	.20	.15	1											
	C2	.33	.13	.20	.14	.27	1										
	C3	.21	.16	.09	.01	.11	.15	1									
	C4	-.01	-.05	-.08	.04	.00	.00	.01	1								
S3	C1	.29	.16	.17	.14	.13	.21	.23	.10	1							
	C2	.21	.27	.23	.11	.25	.12	.25	-.07	.29	1						
	C3	.14	.12	.18	.11	.16	.12	.15	-.09	.32	.33	1					
	C4	.21	.05	.21	.05	.10	.16	.12	-.02	.24	.17	.29	1				
S4	C1	.15	.15	.24	.04	.12	.18	.16	.01	.17	.13	.12	.08	1			
	C2	.10	.13	.14	.19	.10	.17	.15	-.01	.08	.10	.11	.12	.16	1		
	C3	.23	.16	.29	.12	.17	.31	.23	.05	.23	.19	.11	.20	.27	.11	1	
	C4	.19	.11	.14	.11	.09	.14	.13	.04	.15	.15	.14	.05	.07	.17	.15	1

$P(\rho=0) < .01$ if $|r| \geq .13$

Table 40

Correlations Among Items in Test IMC

		S1				S2				S3				S4			
		C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
S1	C1	1															
	C2	.19	1														
	C3	.45	.28	1													
	C4	.05	.03	.06	1												
S2	C1	.21	.22	.18	.13	1											
	C2	.27	.23	.24	.15	.50	1										
	C3	.26	.15	.24	.01	.18	.21	1									
	C4	-.07	.01	-.04	-.07	-.10	-.10	-.09	1								
S3	C1	.03	.09	.06	-.04	.05	-.02	.10	-.04	1							
	C2	.09	.09	.04	.05	.07	.09	.08	-.01	.12	1						
	C3	.07	.20	.19	-.01	.08	.07	.02	-.01	.03	.09	1					
	C4	.09	.01	.09	.02	.06	.09	.10	-.09	.13	.08	.16	1				
S4	C1	.25	.21	.29	.09	.32	.37	.14	-.07	.06	.14	.11	.09	1			
	C2	.04	-.02	-.01	.08	.01	.10	-.05	-.07	.06	-.06	-.02	-.02	.04	1		
	C3	.21	.19	.23	.06	.28	.31	.36	-.06	.16	.08	.15	.12	.25	-.03	1	
	C4	.17	.10	.15	.00	.22	.27	.14	-.13	.07	.01	.03	-.03	.11	.09	.16	1

$P(\rho=0) < .01$ if $|r| \geq .13$

Table 41

Correlations Between Items in Test ID and Test IMC

Test IMC \ Test ID		S1				S2				S3				S4			
		C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
S1	C1	.26	.23	.25	.07	.10	.24	.24	-.04	.19	.13	.18	.11	.22	.08	.27	.12
	C2	.13	.18	.16	.11	.12	.22	.14	.01	.21	.25	.12	.13	.12	.05	.24	.04
	C3	.28	.17	.38	.10	.07	.22	.22	-.02	.21	.23	.18	.13	.22	.10	.26	.15
	C4	.06	.06	.10	.24	.14	.09	.13	.07	.05	.05	.11	.17	.09	.10	.08	.01
S2	C1	.26	.22	.19	.20	.53	.35	.11	.04	.22	.23	.23	.18	.20	.19	.30	.18
	C2	.33	.14	.22	.16	.35	.34	.21	.06	.26	.28	.19	.20	.21	.17	.30	.15
	C3	.14	.14	.09	.05	.12	.10	.25	.04	.09	.16	.10	.07	.21	.10	.28	.13
	C4	-.03	.02	-.09	-.10	.01	-.06	-.06	.01	-.04	-.02	.04	-.08	-.09	.02	-.10	-.14
S3	C1	.09	.10	.12	.02	-.05	-.02	.05	.04	.21	.14	.11	.10	.11	.06	.09	.07
	C2	.05	.04	.02	.03	.05	.01	-.03	.04	.05	.14	.10	.06	.01	.01	.07	.06
	C3	.04	.13	.12	.04	.02	.04	.06	.03	.10	.24	.19	.14	.10	.02	.16	.07
	C4	.09	.01	.03	.04	.07	.09	.07	.07	.10	.01	.00	.16	.08	.01	.11	.00
S4	C1	.27	.14	.25	.05	.21	.26	.10	.08	.25	.18	.16	.18	.33	.14	.26	.11
	C2	.01	.03	.06	.06	.03	.11	.04	-.02	.06	.04	.01	.08	-.07	.21	.06	.17
	C3	.19	.20	.22	.07	.18	.17	.17	.05	.23	.21	.14	.17	.23	.11	.35	.11
	C4	.15	.13	.07	.08	.09	.10	.20	.03	.18	.17	.13	.02	.09	.17	.17	.33

$P(\rho=0) < .01$ if $|r| \geqslant .13$

Table 42

Correlations Among Items in Test IID

		S5				S6				S7				S8			
		C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
S5	C1	1															
	C2	.10	1														
	C3	.06	.13	1													
	C4	.17	.16	.09	1												
S6	C1	.15	.13	.10	.36	1											
	C2	.12	.12	.11	.40	.66	1										
	C3	-.03	-.02	.04	-.01	-.04	.03	1									
	C4	.17	.07	.08	.22	.19	.20	.10	1								
S7	C1	.16	.09	.08	.35	.42	.39	.03	.25	1							
	C2	.16	.09	.13	.25	.22	.24	.08	.23	.23	1						
	C3	.20	.14	.15	.37	.33	.36	.06	.32	.49	.28	1					
	C4	.17	.09	.13	.41	.30	.32	.07	.17	.38	.20	.38	1				
S8	C1	.19	.01	-.02	.08	.02	.05	.03	.08	.15	.15	.18	.09	1			
	C2	.18	.10	.15	.31	.26	.26	.10	.22	.32	.28	.27	.37	.13	1		
	C3	.16	.07	.17	.29	.30	.30	.00	.18	.33	.20	.33	.39	.13	.23	1	
	C4	.18	.13	.10	.30	.29	.26	.02	.18	.27	.21	.28	.30	.11	.35	.27	1

$P(\rho=0) < .01$ if $|r| \geq .13$

Table 43

Correlations Among Items in Test IIMC

		S5				S6				S7				S8			
		C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
S5	C1	1															
	C2	.25	1														
	C3	.13	.28	1													
	C4	.25	.34	.16	1												
S6	C1	.21	.42	.22	.52	1											
	C2	.16	.33	.13	.46	.73	1										
	C3	.26	.17	.11	.20	.24	.21	1									
	C4	.19	.16	.19	.24	.25	.17	.12	1								
S7	C1	.23	.34	.28	.42	.52	.48	.24	.18	1							
	C2	.09	.10	.27	.12	.13	.08	.15	.03	.14	1						
	C3	.25	.41	.36	.40	.51	.47	.18	.23	.61	.15	1					
	C4	.31	.35	.26	.51	.53	.49	.18	.26	.55	.12	.60	1				
S8	C1	.18	.27	.30	.22	.27	.18	.41	.04	.25	.32	.24	.28	1			
	C2	.25	.24	.16	.30	.30	.28	.24	.19	.35	.21	.33	.34	.19	1		
	C3	.16	.28	.26	.32	.42	.42	.27	.27	.41	.06	.43	.28	.20	.22	1	
	C4	.14	.25	.20	.30	.30	.33	.17	.16	.32	.20	.30	.37	.21	.29	.24	1

$P(\rho=0) < .01$ if $|r| \geq .13$

Table 44

Correlations Between Items in Test IID and Test IIMC

Test IIMC \ Test IID		S5				S6				S7				S8			
		C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
S5	C1	.18	.01	.07	.19	.14	.19	.11	.16	.19	.09	.22	.18	.17	.11	.15	.16
	C2	.17	.04	.14	.36	.29	.30	.07	.24	.24	.18	.34	.25	.08	.23	.27	.19
	C3	.14	.10	.18	.23	.14	.15	-.01	.18	.21	.04	.31	.21	.17	.17	.15	.19
	C4	.17	.14	.18	.52	.35	.43	.06	.31	.32	.25	.38	.34	.11	.30	.31	.32
S6	C1	.21	.12	.12	.41	.54	.60	.03	.24	.47	.31	.42	.35	.10	.29	.39	.37
	C2	.21	.06	.14	.39	.54	.55	-.03	.22	.45	.28	.36	.34	.02	.31	.35	.37
	C3	.08	.03	.06	.17	.15	.20	.15	.02	.22	.11	.20	.22	.09	.13	.13	.10
	C4	.10	.10	.02	.26	.14	.14	.11	.42	.16	.11	.23	.18	.06	.13	.21	.17
S7	C1	.16	.14	.08	.38	.31	.33	.08	.18	.55	.26	.49	.34	.09	.32	.31	.35
	C2	.05	.01	.12	.05	-.01	.03	.04	.13	.07	.09	.10	.16	.10	.14	.15	.13
	C3	.21	.19	.12	.41	.34	.34	.04	.28	.43	.28	.51	.37	.13	.31	.37	.33
	C4	.21	.11	.16	.45	.36	.39	.06	.23	.43	.27	.50	.44	.05	.34	.34	.35
S8	C1	.13	.00	.14	.23	.21	.16	.10	.13	.23	.17	.31	.30	.19	.15	.26	.18
	C2	.10	.10	.08	.30	.25	.19	.10	.17	.33	.22	.26	.33	.05	.40	.22	.31
	C3	.21	.18	.18	.30	.31	.36	.02	.24	.25	.18	.39	.30	.07	.19	.42	.29
	C4	.12	.13	.20	.29	.25	.21	.02	.17	.25	.22	.26	.25	.11	.32	.21	.45

$P(\rho=0) < .01$ if $|r| \geq .13$

Table 45

Item Analysis for Test I

	Item		Difficulty	Biserial Correlation	Corrected Biserial Correlation	Reliability	Discriminating Power
Drawing Response	C1	S1	.79	.72	.61	.21	.53
		S2	.81	.58	.47	.16	.37
		S3	.73	.64	.52	.22	.49
		S4	.21	.43	.31	.12	.34
	C2	S1	.72	.60	.48	.20	.53
		S2	.77	.68	.56	.21	.51
		S3	.70	.54	.42	.19	.46
		S4	.09	.16	.06	.03	.06
	C3	S1	.70	.68	.56	.24	.59
		S2	.76	.69	.57	.21	.53
		S3	.69	.58	.46	.21	.52
		S4	.36	.53	.40	.20	.51
	C4	S1	.75	.56	.44	.18	.44
		S2	.14	.55	.44	.12	.35
		S3	.90	.88	.78	.16	.31
		S4	.23	.50	.38	.15	.36
Multiple Choice Response	C1	S1	.89	.77	.66	.15	.31
		S2	.92	.69	.59	.10	.21
		S3	.90	.81	.71	.15	.31
		S4	.15	.39	.28	.09	.27
	C2	S1	.68	.78	.66	.28	.68
		S2	.68	.79	.66	.28	.74
		S3	.91	.65	.55	.10	.19
		S4	.28	-.08	-.20	-.03	-.02
	C3	S1	.83	.36	.25	.09	.23
		S2	.83	.30	.19	.08	.16
		S3	.87	.42	.32	.09	.21
		S4	.59	.30	.18	.12	.29
	C4	S1	.65	.68	.55	.25	.60
		S2	.20	.24	.12	.07	.19
		S3	.87	.78	.67	.16	.33
		S4	.39	.48	.35	.18	.48

N = 432 $\quad$ S = 4.95

X = 19.95 $\quad$ KR20 = .81

$S^2 = 24.5$ $\quad$ $SE_{MEAS} = 2.15$

Table 46

Item Analysis for Test II

	Item		Difficulty	Biserial Correlation	Corrected Biserial Correlation	Reliability	Discriminating Power
Drawing Response	C5	S1	.96	.73	.66	.06	.12
		S2	.97	.52	.46	.04	.08
		S3	.95	.52	.45	.06	.12
		S4	.59	.80	.70	.31	.79
	C6	S1	.74	.79	.70	.26	.62
		S2	.73	.82	.72	.27	.67
		S3	.98	.31	.25	.01	.03
		S4	.47	.57	.47	.23	.59
	C7	S1	.82	.90	.82	.24	.48
		S2	.66	.58	.49	.21	.51
		S3	.81	.94	.86	.26	.55
		S4	.73	.79	.70	.26	.60
	C8	S1	.94	.42	.34	.05	.12
		S2	.79	.74	.65	.22	.49
		S3	.75	.74	.65	.24	.55
		S4	.43	.69	.60	.27	.77
Multiple Choice Response	C5	S1	.88	.62	.34	.12	.28
		S2	.73	.75	.65	.25	.55
		S3	.84	.60	.51	.15	.33
		S4	.60	.84	.74	.32	.85
	C6	S1	.71	.98	.89	.33	.83
		S2	.78	.95	.86	.28	.68
		S3	.96	.79	.72	.07	.11
		S4	.73	.56	.47	.18	.49
	C7	S1	.76	.93	.84	.29	.63
		S2	.94	.46	.39	.05	.12
		S3	.74	.96	.87	.31	.67
		S4	.70	.94	.85	.33	.78
	C8	S1	.93	.82	.74	.11	.22
		S2	.79	.73	.64	.21	.45
		S3	.82	.83	.75	.22	.48
		S4	.69	.69	.59	.24	.61

$N = 432$ $S = 6.45$

$X = 24.89$ $KR20 = .91$

$S^2 = 41.6$ $SE_{MEAS} = 2.15$

APPENDIX 4

Participating Schools and Teachers and Their Instructions

Schools Participating in the Study

Edmonton Public School Board	Classes
Afton Elementary School	3 Grade 5
Allendale Elementary Junior High School	1 Grade 5, 1 Grade 6
Athlone Elementary School	2 Grade 6
Harry Ainlay Composite High School	3 Grade 10
Vernon Barford Junior High School	3 Grade 7, 3 Grade 8, 3 Grade 9

Edmonton Separate School Board	
H.E. Beriault Separate School	2 Grade 7
St. Brendan Separate School	3 Grade 9
St. Edmund Separate School	2 Grade 8
St. Mary's Separate School	3 Grade 10
St. Nicholas Separate School	1 Grade 5, 1 Grade 6
St. Patrick Separate School	2 Grade 5, 2 Grade 6
St. Pius X Separate School	2 Grade 7
St. Vincent Separate School	1 Grade 7

Teachers Participating in the Study

Mr. Anderson
Mr. Biederman
Mr. Clintberg
Mr. Curial
Mr. Dombronski
Miss Dunningan
Mr. Fletcher
Miss Hihn
Mr. Holland
Mr. Hubick
Mrs. Hughes
Mr. Kostyshyn
Mrs. Kutney
Mr. Landerville
Mr. Lee
Mrs. Lefleur
Mrs. Lerminaux
Mr. Peters
Miss Pokinghorn
Mr. Pomfrey
Mr. Pookey
Mr. Roles
Mr. Syrnyk
Mr. Walker
Mr. Weathernick

Your participation in this research is much appreciated. Should you have any questions (or problems) concerning the study please contact me at 432-3760 during the day or at 435-7693 in the evening.

Test II should be administered immediately after the completion of your unit on geometry. It would be appreciated if you could contact me as soon as you become aware of your completion date.

Thank you again for your assistance and participation.

Dale R. Drost

It would be appreciated if you could assist in collecting the following data on each student involved in the study.

1. The most recent IQ score available. The name of the test would be of assistance if it is available.

2. The mathematics final grade from the previous year. If a system wide test was administered this would be ideal, otherwise the regular final grade would be adequate.

3. The mathematics grade received on the last major test prior to Test I.

4. Achievement data on the unit following Test I, including a copy of any tests administered.

The researcher is prepared to assist in any way he can in the collection of the above data. If the information must be transferred from cumulative record cards or from some other source, this could be done by the researcher if the regular teacher so wishes.

One of the main objectives of this study is to determine the predictive validity of the ability to section selected solids on achievement in geometry. To help answer this question it is necessary to collect information on the types of geometry being taught, the areas of special emphasis and the methods used to teach the material. Stated differently, it is important to know what is happening in each classroom before the above question can be answered. We would appreciate it a great deal if you could keep a brief daily record of the events occurring in your classroom.

The following categories are suggested:

1. Objectives of lesson
2. Points of emphasis
3. Materials used
4. Teaching method
5. Other comments

An example of a possible record is included on the following page.

Thank you for your cooperation.

Class 7A Date Nov 8

1. Objectives:

The students will be able to

1) state definitions of reflection, rotation and translation

2) select convex figures from a list

* you might wish to enumerate the objectives from the system materials which you might be using.

2. Particular points of emphasis:

The emphasis was on doing the motions

or

The emphasis was on learning the definitions

3. Materials used:

Solid geometry models

Compass, protractor & ruler.

4. Teaching method:

1) Motions were illustrated on overhead projector and students did examples at their seats

or 2) The material was presented by lecture only.

5. Other comments:

The slow kids had a lot of trouble.

or Everyone can finally do reflections.

Class ____________________ Date _________

1. Objectives:

2. Particular points of emphasis:

3. Materials used:

4. Teaching methods:

5. Other comments:

APPENDIX 5

Supplementary Tables

The tables on the following pages are the results of principal axis factor analyses under varimax rotation for each of grades 5 and 6 combined, grades 7 and 8 combined, grades 9 and 10 combined, each sex, and each ability level. The four factor solution is presented for each of these groups for Test I and the five factor solution for Test II. Decimal points have been omitted from entries within the tables. Entries with an absolute value greater than or equal to .30 have been underlined for emphasis. The blank spaces in Tables 57 and 62 indicate that the item was deleted from the analysis since all subjects answered the item correctly and hence its variance was zero.

Table 47
Principal Axis Factoring Under Varimax
Rotation for Test I - Grades 5,6 Combined

	Variable		Factor 1	Factor 2	Factor 3	Factor 4	Communalities
Drawing Responses	S1	C1	32	37	05	10	25
		C2	06	18	36	00	16
		C3	11	41	04	15	21
		C4	27	16	-10	12	12
	S2	C1	57	-18	24	-02	42
		C2	44	24	-01	12	26
		C3	05	39	02	09	16
		C4	03	-18	-14	20	10
	S3	C1	14	07	23	58	41
		C2	06	21	50	27	36
		C3	04	07	40	23	22
		C4	08	12	-05	44	22
	S4	C1	18	22	24	09	14
		C2	17	24	00	16	11
		C3	32	31	19	33	34
		C4	44	15	-12	04	23
Multiple Choice Responses	S1	C1	23	46	24	-15	34
		C2	15	20	30	18	18
		C3	11	63	32	-02	51
		C4	01	04	04	09	01
	S2	C1	69	-17	27	-01	58
		C2	48	15	25	03	32
		C3	31	17	35	-03	25
		C4	-19	-14	36	-10	19
	S3	C1	-05	-04	09	46	23
		C2	04	00	39	15	18
		C3	-04	26	26	17	17
		C4	28	27	-11	33	27
	S4	C1	29	17	16	34	25
		C2	07	01	-28	18	11
		C3	36	16	25	23	27
		C4	40	19	-04	04	20
Eigen values			4.33	1.28	1.16	1.02	
Variance			2.49	1.91	1.83	1.56	
% Total Variance			7.79	5.97	5.71	4.88	
% Common Variance			32.0	24.5	23.5	20.0	

Sum of Communalities = 7.79
Total Variance Accounted for = 24.3%

Table 48

Principal Axis Factoring Under Varimax

Rotation for Test I - Grades 7,8 Combined

	Variable		Factor 1	2	3	4	Communalities
Drawing Responses	S1	C1	36	43	22	14	39
		C2	35	12	31	23	29
		C3	47	35	24	07	41
		C4	01	35	02	09	13
	S2	C1	08	60	27	-02	44
		C2	16	49	14	18	31
		C3	35	14	13	24	22
		C4	01	04	-17	03	03
	S3	C1	31	35	21	28	34
		C2	11	36	52	10	43
		C3	14	35	51	06	40
		C4	14	23	49	-19	35
	S4	C1	52	05	04	16	30
		C2	06	19	-01	35	16
		C3	60	30	01	-01	45
		C4	22	28	-08	27	21
Multiple Choice Responses	S1	C1	56	15	13	09	37
		C2	22	01	33	04	16
		C3	61	19	07	-06	42
		C4	01	24	02	00	06
	S2	C1	26	72	08	02	60
		C2	28	68	-04	-08	55
		C3	60	-10	08	00	37
		C4	-25	-15	24	15	16
	S3	C1	38	-16	28	03	25
		C2	-01	09	00	-33	12
		C3	02	10	22	-13	08
		C4	08	-04	01	-32	11
	S4	C1	47	33	-09	-11	35
		C2	12	15	-16	32	16
		C3	59	20	08	-22	44
		C4	28	25	-13	17	19
Eigen values			5.73	1.45	1.13	.90	
Variance			3.56	3.10	1.55	.99	
% Total Variance			11.1	9.68	4.86	3.08	
% Common Variance			38.7	33.7	16.9	10.7	

Sum of Communalities = 9.20

Total Variance Accounted for = 28.7%

Table 49
Principal Axis Factoring Under Varimax
Rotation for Test I - Grades 9,10 Combined

	Variable		Factor 1	2	3	4	Communalities
Drawing Responses	S1	C1	27	22	20	-20	20
		C2	-31	-08	-06	15	13
		C3	14	-12	59	15	40
		C4	18	-15	09	11	07
	S2	C1	59	-10	-02	00	36
		C2	50	-02	14	-07	27
		C3	00	32	-01	-04	10
		C4	22	11	-08	-14	09
	S3	C1	11	43	15	10	23
		C2	03	38	-03	08	15
		C3	-06	14	02	54	32
		C4	10	06	36	45	35
	S4	C1	40	-11	40	05	34
		C2	25	-05	10	35	20
		C3	27	11	12	-01	10
		C4	-04	22	07	36	18
Multiple Choice Responses	S1	C1	10	29	41	-10	27
		C2	10	65	36	-11	57
		C3	-05	22	59	-05	41
		C4	20	-27	14	23	18
	S2	C1	60	13	07	13	39
		C2	65	31	09	03	53
		C3	23	01	07	-04	06
		C4	13	05	00	-21	06
	S3	C1	12	31	-05	22	16
		C2	14	-01	-04	11	03
		C3	00	40	10	26	24
		C4	12	11	02	-03	03
	S4	C1	41	05	47	10	40
		C2	09	00	-10	26	08
		C3	36	18	07	12	18
		C4	03	32	-28	31	28
Eigen values			3.34	1.60	1.26	1.13	
Variance			2.46	1.78	1.77	1.34	
% Total Variance			7.67	5.55	5.52	4.18	
% Common Variance			33.5	24.2	24.1	18.2	

Sum of Communalities = 7.34
Total Variance Accounted for = 22.9%

Table 50

Principal Axis Factoring Under

Varimax Rotation for Test I - Males

	Variable		Factor 1	Factor 2	Factor 3	Factor 4	Communalities
Drawing Responses	S1	C1	28	29	30	01	25
		C2	09	43	22	14	26
		C3	15	28	18	20	18
		C4	23	01	06	32	16
	S2	C1	51	08	10	27	35
		C2	56	17	11	12	37
		C3	01	45	11	17	24
		C4	21	-11	00	-18	09
	S3	C1	27	20	41	07	28
		C2	05	23	41	12	24
		C3	-06	13	56	27	41
		C4	09	06	48	20	28
	S4	C1	33	39	07	06	27
		C2	05	11	07	39	17
		C3	47	30	21	13	37
		C4	13	07	07	18	06
Multiple Choice Responses	S1	C1	15	68	08	-02	50
		C2	31	11	04	02	11
		C3	10	63	02	-02	41
		C4	09	05	06	37	15
	S2	C1	69	10	10	35	61
		C2	55	28	13	28	48
		C3	20	49	-04	-04	28
		C4	-20	04	07	01	05
	S3	C1	-12	01	37	07	15
		C2	05	04	22	-12	07
		C3	10	-06	41	-02	19
		C4	29	03	34	-13	22
	S4	C1	44	33	21	-03	35
		C2	-03	-08	-03	39	16
		C3	40	23	37	09	36
		C4	20	26	-01	24	17

	1	2	3	4
Eigen values	5.07	1.21	1.14	.78
Variance	2.71	2.42	1.85	1.23
% Total Variance	8.46	7.55	5.78	3.83
% Common Variance	33.0	29.5	22.6	14.9

Sum of Communalities = 8.20

Total Variance Accounted for = 25.6%

Table 51
Principal Axis Factoring Under
Varimax Rotation for Test I - Females

	Variable		Factor 1	2	3	4	Communalities
Drawing Responses	S1	C1	37	33	01	23	30
		C2	22	30	09	-02	15
		C3	61	20	06	04	41
		C4	07	19	-01	18	07
	S2	C1	04	62	08	-04	40
		C2	35	29	-12	24	27
		C3	20	05	26	33	22
		C4	-11	-01	09	21	06
	S3	C1	31	20	18	27	24
		C2	28	36	34	19	36
		C3	28	32	09	11	20
		C4	30	13	08	09	12
	S4	C1	31	-01	24	07	16
		C2	11	14	05	41	20
		C3	34	07	36	22	30
		C4	16	18	11	54	35
Multiple Choice Responses	S1	C1	48	10	05	16	27
		C2	44	17	29	-05	31
		C3	64	10	17	12	46
		C4	01	21	-06	07	05
	S2	C1	17	66	16	09	50
		C2	23	50	16	26	40
		C3	07	05	65	15	45
		C4	-08	04	-02	-30	10
	S3	C1	14	-07	29	14	13
		C2	06	17	18	-06	07
		C3	39	-12	18	00	20
		C4	13	-17	06	-01	05
	S4	C1	39	27	09	11	24
		C2	10	03	-19	40	21
		C3	22	10	59	-01	41
		C4	-01	13	25	47	30
Eigen values			4.92	1.19	.98	.87	
Variance			2.64	2.06	1.67	1.59	
% Total Variance			8.26	6.43	5.22	4.96	
% Common Variance			33.2	25.9	21.0	20.0	

Sum of Communalities = 7.96
Total Variance Accounted for = 24.9%

Table 52
Principal Axis Factoring Under
Varimax Rotation for Test I - Low Ability

	Variable		Factor 1	2	3	4	Communalities
Drawing Responses	S1	C1	29	29	-13	24	24
		C2	08	27	-03	21	12
		C3	28	20	01	30	21
		C4	32	08	07	-14	13
	S2	C1	53	-02	-12	12	31
		C2	38	23	16	-19	26
		C3	-04	57	-01	-06	33
		C4	-04	-02	15	00	02
	S3	C1	25	40	-10	22	28
		C2	34	43	-19	25	39
		C3	39	27	-25	12	30
		C4	39	08	07	07	17
	S4	C1	13	21	43	15	26
		C2	06	25	19	-31	20
		C3	34	37	32	01	35
		C4	08	35	03	02	13
Multiple Choice Responses	S1	C1	12	43	20	21	28
		C2	17	24	08	24	15
		C3	10	36	15	25	22
		C4	33	02	01	-27	19
	S2	C1	63	15	16	10	45
		C2	49	25	16	08	34
		C3	06	39	24	19	25
		C4	-17	00	-32	05	13
	S3	C1	-04	13	04	26	09
		C2	09	09	09	29	11
		C3	10	16	36	05	17
		C4	-08	-04	38	02	15
	S4	C1	25	06	39	37	35
		C2	08	-03	-03	-37	14
		C3	14	31	27	25	26
		C4	30	43	-02	-03	27
Eigen values			4.25	1.18	1.05	.77	
Variance			2.36	2.31	1.30	1.29	
% Total Variance			7.36	7.21	4.06	4.03	
% Common Variance			32.5	31.8	17.9	17.8	

Sum of Communalities = 7.25
Total Variance Accounted for = 22.7%

Table 53
Principal Axis Factoring Under
Varimax Rotation for Test I - Average Ability

	Variable		Factor 1	Factor 2	Factor 3	Factor 4	Communalities
Drawing Responses	S1	C1	44	28	09	-09	29
		C2	16	10	20	-38	22
		C3	49	13	16	-26	36
		C4	02	28	08	13	10
	S2	C1	-05	62	03	-12	40
		C2	13	56	-02	-04	33
		C3	41	09	23	-12	24
		C4	01	11	05	48	25
	S3	C1	20	19	49	19	36
		C2	00	07	58	-23	39
		C3	-02	06	51	-04	26
		C4	19	-05	40	13	22
	S4	C1	33	13	03	-10	14
		C2	02	20	03	-12	05
		C3	55	10	16	14	36
		C4	26	11	-03	08	09
Multiple Choice Responses	S1	C1	48	15	03	-31	35
		C2	06	25	44	09	27
		C3	63	12	27	-22	52
		C4	09	17	09	11	06
	S2	C1	07	78	07	-06	62
		C2	30	65	07	03	51
		C3	43	00	08	00	19
		C4	-37	07	16	-13	18
	S3	C1	23	-16	33	21	23
		C2	-01	-07	20	34	16
		C3	03	06	63	-06	41
		C4	19	11	23	11	12
	S4	C1	26	36	16	08	23
		C2	11	-04	00	-01	01
		C3	42	20	28	07	30
		C4	35	14	02	17	18
Eigen values			4.34	1.65	1.38	1.01	
Variance			2.72	2.42	2.19	1.05	
% Total Variance			8.50	7.55	6.84	3.29	
% Common Variance			32.5	28.8	26.1	12.6	

Sum of Communalities = 8.38
Total Variance Accounted for = 26.2%

Table 54
Principal Axis Factoring Under
Varimax Rotation for Test I - High Ability

	Variable		Factor 1	2	3	4	Communalities
Drawing Responses	S1	C1	48	06	07	15	27
		C2	20	35	16	01	19
		C3	63	-09	02	22	46
		C4	09	07	22	15	08
	S2	C1	06	37	24	39	34
		C2	68	26	11	05	54
		C3	09	25	29	05	16
		C4	-01	12	00	-02	02
	S3	C1	03	01	01	34	12
		C2	-16	28	33	30	31
		C3	-06	03	09	19	05
		C4	29	-10	-03	32	20
	S4	C1	37	28	-13	29	31
		C2	17	03	43	19	25
		C3	34	45	07	13	34
		C4	04	03	52	01	28
Multiple Choice Responses	S1	C1	48	09	16	-26	33
		C2	31	54	-05	-05	39
		C3	53	21	13	27	42
		C4	08	00	04	36	14
	S2	C1	17	50	29	40	52
		C2	20	39	30	30	37
		C3	02	33	20	-18	18
		C4	03	-01	-27	-02	08
	S3	C1	01	-01	08	16	03
		C2	-17	08	16	12	08
		C3	-04	25	-07	-09	08
		C4	-03	-06	-22	30	14
	S4	C1	24	32	15	28	26
		C2	07	-20	42	16	25
		C3	00	46	04	13	23
		C4	07	13	51	-15	31
Eigen values			3.95	1.54	1.18	1.02	
Variance			2.33	2.08	1.72	1.56	
% Total Variance			2.29	6.51	3.38	4.87	
% Common Variance			30.3	27.1	22.4	20.3	

Sum of Communalities = 7.70
Total Variance Accounted for = 24.1%

Table 55
Principal Axis Factoring Under Varimax
Rotation for Test II - Grades 5,6 Combined

	Variable		Factor 1	2	3	4	5	Communalities
Drawing Responses	S5	C1	04	03	24	18	21	13
		C2	12	06	16	04	16	07
		C33	15	-02	08	16	07	06
		C4	42	16	31	10	28	38
	S6	C1	65	22	12	12	-01	50
		C2	69	30	17	04	03	60
		C3	-21	-01	11	-09	14	08
		C4	07	19	04	00	60	40
	S7	C1	35	62	21	03	03	56
		C2	12	14	42	-01	03	21
		C3	30	46	16	23	33	49
		C4	17	42	33	22	07	36
	S8	C1	-14	08	31	29	05	21
		C2	09	28	47	00	06	32
		C3	21	35	22	29	18	33
		C4	21	15	46	17	-01	31
Multiple Choice Responses	S5	C1	10	14	19	28	20	18
		C2	49	10	15	19	31	41
		C3	00	45	-10	38	29	44
		C4	44	18	43	11	25	48
	S6	C1	68	36	33	09	13	73
		C2	69	36	29	-03	02	69
		C3	30	09	11	55	02	41
		C4	-03	04	02	-08	61	38
	S7	C1	28	72	28	09	08	68
		C2	-11	14	19	47	-13	31
		C3	19	64	31	14	26	62
		C4	35	53	40	05	15	59
	S8	C1	21	09	05	73	-04	59
		C2	07	22	41	17	09	25
		C3	42	35	06	26	44	56
		C4	19	01	47	19	-01	29
Eigen values			8.28	1.37	1.24	.98	.75	
Variance			3.52	3.15	2.39	1.89	1.66	
% Total Variance			11.0	9.85	7.48	5.90	5.18	
% Common Variance			27.9	25.0	19.0	15.0	13.1	

Sum of Communalities = 12.6
Total Variance Accounted for = 39.4%

Table 56
Principal Axis Factoring Under Varimax
Rotation for Test II - Grades 7,8 Combined

	Variable		Factor 1	2	3	4	5	Communalities
Drawing Responses	S5	C1	33	-03	05	28	00	19
		C2	03	01	00	25	-08	07
		C3	11	-03	-04	26	48	31
		C4	35	34	26	25	04	36
	S6	C1	66	16	09	13	-30	57
		C2	77	03	11	-08	02	61
		C3	-05	11	03	-05	37	16
		C4	17	07	40	06	18	23
	S7	C1	41	30	19	23	-08	35
		C2	22	20	23	13	10	17
		C3	41	43	27	15	04	45
		C4	36	08	18	35	28	37
	S8	C1	12	-02	41	-12	-01	20
		C2	18	23	04	66	14	54
		C3	56	02	29	25	21	50
		C4	29	27	12	38	-02	31
Multiple Choice Responses	S5	C1	23	28	01	-23	36	32
		C2	03	49	27	00	08	32
		C3	-03	17	63	19	-05	27
		C4	50	29	06	13	15	37
	S6	C1	71	28	17	03	06	61
		C2	68	30	-06	13	16	60
		C3	07	20	08	-18	56	40
		C4	29	15	37	-08	14	26
	S7	C1	15	70	10	03	08	53
		C2	-05	-05	29	17	50	37
		C3	32	68	11	09	14	61
		C4	43	47	09	23	05	46
	S8	C1	07	21	58	13	13	42
		C2	19	45	-05	25	19	34
		C3	31	19	18	03	31	26
		C4	01	43	21	45	19	47
Eigen values			7.16	1.68	1.19	1.12	1.04	
Variance			4.04	2.91	1.88	1.70	1.66	
% Total Variance			12.6	9.10	5.88	5.30	5.19	
% Common Variance			33.1	23.9	15.4	13.9	13.6	

Sum of Communalities = 12.2
Total Variance Accounted for = 38.1%

Table 57

Principal Axis Factoring Under Varimax

Rotation for Test II - Grades 9,10 Combined

	Variable		Factor 1	2	3	4	5	Communalities
Drawing Responses	S5	C1	-	-	-	-	-	-
		C2	-07	19	05	-04	82	71
		C3	-02	00	-04	09	81	66
		C4	25	44	32	09	21	41
	S6	C1	10	73	-05	26	04	62
		C2	04	72	06	19	02	56
		C3	69	16	-04	-05	01	50
		C4	-01	24	32	-18	-05	19
	S7	C1	17	16	42	19	-05	27
		C2	15	34	20	14	03	20
		C3	17	10	71	00	-06	54
		C4	31	36	24	25	00	34
	S8	C1	-01	-05	43	-05	-05	19
		C2	39	27	17	07	-05	27
		C3	-02	15	00	36	-05	15
		C4	20	05	25	44	01	30
Multiple Choice Responses	S5	C1	49	-11	02	02	06	25
		C2	36	14	12	-10	-04	17
		C3	01	-30	-06	21	39	29
		C4	27	28	43	07	25	40
	S6	C1	27	48	25	32	04	47
		C2	08	49	01	51	-08	51
		C3	56	07	-04	12	-02	33
		C4	35	15	05	09	-08	16
	S7	C1	29	19	47	46	05	55
		C2	17	-24	20	17	06	16
		C3	-09	07	63	26	09	49
		C4	41	-06	44	31	05	46
	S8	C1	57	03	35	01	-01	44
		C2	60	00	07	20	01	40
		C3	16	29	07	49	05	36
		C4	-02	05	01	60	02	36

	1	2	3	4	5
Eigen values	5.64	1.88	1.64	1.42	1.13
Variance	2.85	2.64	2.51	2.09	1.63
% Total Variance	9.18	8.50	8.11	6.73	5.26
% Common Variance	22.3	22.5	21.5	17.8	13.9

Sum of Communalities = 11.7

Total Variance Accounted for = 37.8%

Table 58
Principal Axis Factoring Under Varimax
Rotation for Test II - Males

	Variable		Factor 1	2	3	4	5	Communalities
Drawing Responses	S5	C1	24	09	12	28	-05	16
		C2	12	17	-10	04	43	24
		C3	09	10	18	09	51	31
		C4	31	53	06	22	19	46
	S6	C1	63	25	09	11	02	48
		C2	75	24	06	05	07	64
		C3	-07	-07	-07	15	-06	04
		C4	19	19	03	55	08	38
	S7	C1	52	42	00	02	18	48
		C2	25	11	09	16	04	11
		C3	42	28	28	17	27	44
		C4	34	33	22	26	11	35
	S8	C1	13	03	07	11	-18	07
		C2	19	48	10	19	12	32
		C3	39	25	07	30	20	34
		C4	18	44	04	23	-01	28
Multiple Choice Responses	S5	C1	11	23	06	07	-27	15
		C2	21	39	14	17	-02	25
		C3	03	27	51	12	27	41
		C4	52	37	01	19	09	45
	S6	C1	75	28	10	13	-11	68
		C2	67	32	12	07	-14	60
		C3	23	-01	30	-06	-06	15
		C4	14	27	-04	55	04	40
	S7	C1	44	64	-02	-06	05	61
		C2	-05	19	51	-10	-10	32
		C3	34	58	13	07	11	49
		C4	42	59	21	12	-03	58
	S8	C1	16	08	69	10	00	52
		C2	09	52	17	10	04	32
		C3	49	27	04	23	31	46
		C4	16	48	13	-01	05	27
Eigen values			8.12	1.19	1.00	.75	.70	
Variance			4.26	3.71	1.47	1.31	1.01	
% Total Variance			13.3	11.6	4.59	4.09	3.16	
% Common Variance			36.2	31.6	12.5	11.1	8.61	

Sum of Communalities = 11.8
Total Variance Accounted for = 36.7%

Table 59
Principal Axis Factoring Under Varimax
Rotation for Test II - Females

	Variable		Factor 1	2	3	4	5	Communalities
Drawing Responses	S5	C1	08	14	19	06	28	14
		C2	07	12	13	03	14	06
		C3	13	06	-02	02	36	15
		C4	42	25	26	19	08	35
	S6	C1	74	15	04	06	03	57
		C2	71	08	10	09	04	53
		C3	-06	21	04	34	-01	17
		C4	08	13	57	00	14	37
	S7	C1	33	43	22	32	-15	46
		C2	20	43	21	03	22	32
		C3	25	41	47	32	-03	55
		C4	24	41	02	42	13	42
	S8	C1	-12	05	18	32	21	19
		C2	09	53	08	12	19	34
		C3	33	17	06	35	27	33
		C4	23	52	07	08	30	42
Multiple Choice Responses	S5	C1	12	12	33	37	07	28
		C2	37	08	39	20	26	40
		C3	11	06	41	22	19	27
		C4	32	32	40	09	23	43
	S6	C1	67	33	30	10	18	69
		C2	75	36	14	-03	15	73
		C3	16	12	12	63	04	45
		C4	05	07	49	11	-02	26
	S7	C1	29	48	32	34	00	52
		C2	-12	09	11	23	49	33
		C3	36	45	48	22	12	63
		C4	35	52	32	19	03	54
	S8	C1	15	08	11	59	22	44
		C2	10	51	09	28	12	37
		C3	33	09	21	33	28	34
		C4	12	39	16	10	41	37
Eigen values			8.43	1.51	.85	.84	.77	
Variance			3.51	3.00	2.30	2.23	1.37	
% Total Variance			11.0	9.37	7.18	6.98	4.27	
% Common Variance			28.3	24.2	18.5	18.0	11.0	

Sum of Communalities = 12.4
Total Variance Accounted for = 38.8%

Table 60
Principal Axis Factoring Under Varimax
Rotation for Test II - Low Ability

	Variable		Factor 1	2	3	4	5	Communalities
Drawing Responses	S5	C1	28	-03	10	05	10	10
		C2	15	08	11	-17	-03	07
		C3	04	17	22	00	-26	15
		C4	58	19	11	-02	12	40
	S6	C1	30	51	-17	10	01	39
		C2	19	62	-09	15	14	47
		C3	04	05	14	51	-01	28
		C4	34	19	19	-02	47	41
	S7	C1	34	33	04	23	01	29
		C2	47	19	16	06	-04	29
		C3	45	25	29	22	18	44
		C4	33	35	19	24	-10	33
	S8	C1	10	-14	26	21	13	16
		C2	54	02	06	24	-15	38
		C3	11	51	17	06	01	30
		C4	42	20	-10	08	04	23
Multiple Choice Responses	S5	C1	14	06	-06	49	34	39
		C2	46	28	31	07	06	39
		C3	14	-02	54	-04	15	33
		C4	54	27	02	05	10	38
	S6	C1	40	68	06	01	16	65
		C2	40	63	-15	-06	00	59
		C3	08	24	21	46	-07	32
		C4	07	20	05	07	62	44
	S7	C1	63	32	14	03	-01	52
		C2	07	00	48	12	-04	25
		C3	57	22	19	00	19	45
		C4	59	32	00	12	27	53
	S8	C1	18	16	54	25	-10	42
		C2	41	11	15	43	01	38
		C3	14	59	23	-02	10	43
		C4	50	14	18	-01	00	30
Eigen values			7.15	1.55	.97	.95	.83	
Variance			4.24	3.17	1.57	1.34	1.12	
% Total Variance			13.3	9.92	4.91	4.19	3.49	
% Common Variance			37.1	27.7	13.7	11.7	9.76	

Sum of Communalities = 11.4
Total Variance Accounted for = 35.8%

Table 61
Principal Axis Factoring Under Varimax
Rotation for Test II - Average Ability

	Variable		Factor 1	2	3	4	5	Communalities
Drawing Responses	S5	C1	22	49	07	-03	-10	30
		C2	03	34	14	-14	-02	16
		C3	15	36	12	00	-15	19
		C4	46	12	16	12	-03	27
	S6	C1	67	21	12	04	-26	58
		C2	72	10	14	01	-15	57
		C3	-04	-07	20	-05	29	13
		C4	11	34	17	-04	23	21
	S7	C1	61	27	31	15	34	68
		C2	10	07	31	-03	16	13
		C3	63	40	02	10	22	61
		C4	39	28	32	16	05	36
	S8	C1	02	27	03	13	00	09
		C2	35	27	62	-06	04	58
		C3	35	33	10	09	10	26
		C4	18	20	47	19	06	33
Multiple Choice Responses	S5	C1	30	24	-24	44	11	41
		C2	47	10	-11	07	12	26
		C3	21	54	-08	23	18	43
		C4	53	20	24	15	04	40
	S6	C1	69	03	22	19	14	59
		C2	70	13	29	16	01	62
		C3	16	-02	04	75	-05	60
		C4	-06	33	29	01	22	25
	S7	C1	57	08	17	22	57	73
		C2	04	-07	22	37	12	20
		C3	57	37	13	08	45	69
		C4	60	19	24	14	34	58
	S8	C1	19	16	07	80	-04	71
		C2	29	-03	60	05	08	45
		C3	34	63	-01	10	12	54
		C4	22	18	51	31	-01	44
Eigen values			8.39	1.56	1.34	1.11	.93	
Variance			5.37	2.45	2.20	2.02	1.27	
% Total Variance			16.8	7.65	6.87	6.32	3.98	
% Common Variance			40.3	18.4	16.5	15.2	9.57	

Sum of Communalities = 13.3
Total Variance Accounted for = 41.6%

Table 62
Principal Axis Factoring Under Varimax
Rotation for Test II - High Ability

	Variable		Factor 1	2	3	4	5	Communalities
Drawing Responses	S5	C1	-	-	-	-	-	
		C2	21	-04	35	18	12	21
		C3	07	20	24	03	37	24
		C4	39	25	16	34	15	37
	S6	C1	89	-02	17	03	04	83
		C2	88	08	06	11	-09	80
		C3	-04	-05	-08	-06	18	05
		C4	03	-04	14	07	58	36
	S7	C1	29	19	24	54	01	47
		C2	29	10	19	00	08	14
		C3	11	08	-02	63	26	48
		C4	-02	30	28	44	-05	37
	S8	C1	-06	-03	-02	13	-02	02
		C2	04	12	34	05	22	18
		C3	13	43	26	26	07	34
		C4	26	33	39	16	07	35
Multiple Choice Responses	S5	C1	-01	09	63	10	00	42
		C2	17	35	20	11	18	23
		C3	11	59	09	10	21	43
		C4	39	31	20	10	19	34
	S6	C1	86	31	05	12	03	86
		C2	87	13	05	01	05	78
		C3	-	-	-	-	-	-
		C4	13	15	13	09	52	33
	S7	C1	17	25	18	60	-03	48
		C2	-04	54	28	-22	04	43
		C3	32	44	17	45	06	54
		C4	15	37	49	40	-12	58
	S8	C1	00	48	-02	01	-25	30
		C2	11	27	57	03	10	42
		C3	31	07	07	26	26	24
		C4	16	09	54	00	05	33
Eigen values			6.42	2.24	1.24	1.11	.90	
Variance			4.05	2.29	2.28	2.07	1.23	
% Total Variance			13.5	7.63	7.60	6.90	4.10	
% Common Variance			34.0	19.2	19.1	17.4	10.3	

Sum of Communalities = 11.9
Total Variance Accounted for = 39.7%